

# Chapter 3

## Planning with Deterministic Models

3.3. Backward Search

3.4. Plan-Space Search

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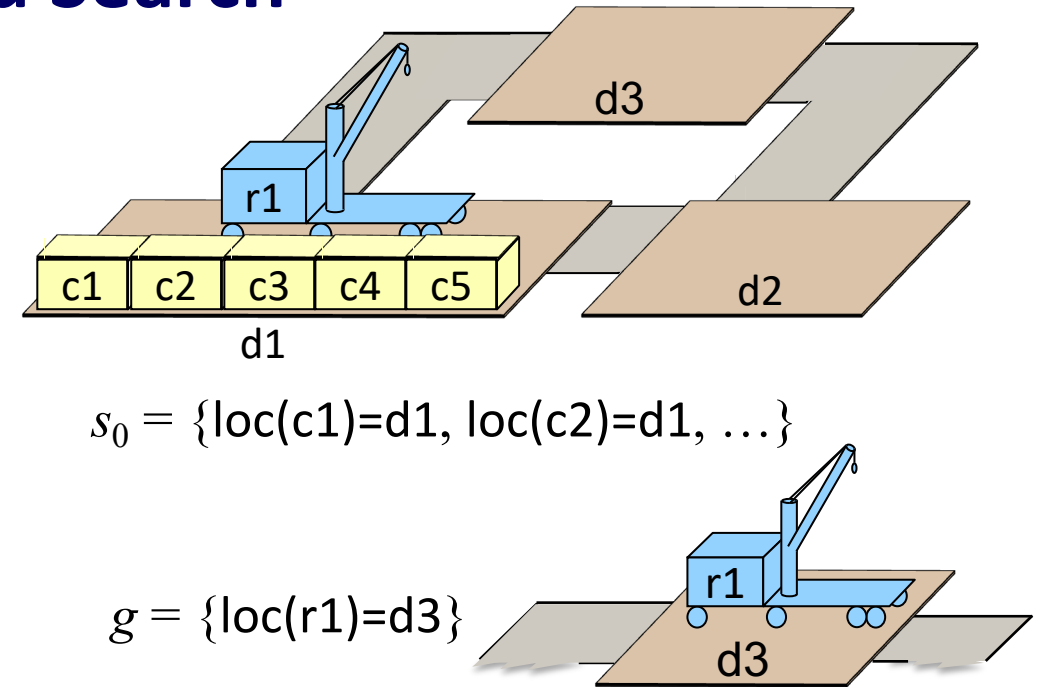
with contributions from

[Mark “mak” Roberts](#)



## 3.3. Backward Search

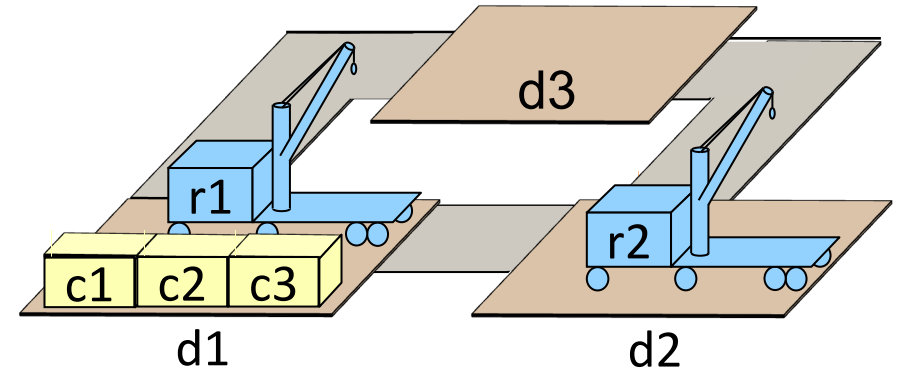
- Forward search: forward from initial state
  - ▶ In state  $s$ , choose applicable action  $a$
  - ▶ Compute state transition  $s' = \gamma(s, a)$
- Backward search: backward from the goal
  - ▶ For goal  $g$ , choose *relevant* action  $a$ 
    - A possible “last action” before the goal
    - Sometimes this has a lower branching factor
- Compute *inverse* state transition  $g' = \gamma^{-1}(g, a)$ 
  - ▶  $g' =$  properties a state  $s'$  should satisfy in order for  $\gamma(s', a)$  to satisfy  $g$
- Equivalently, if  $S_g = \{\text{all states that satisfy } g\}$  then
  - ▶  $S_{g'} = \{\text{all states } s \text{ such that } \gamma(s, a) \in S_g\}$



- ▶ Forward: 7 applicable actions
  - five load actions, two move actions
- ▶ Backward:  $g = \{\text{loc}(r1)=d3\}$ 
  - two relevant actions:  
 $\text{move}(r1, d1, d3)$ ,  $\text{move}(r1, d2, d3)$

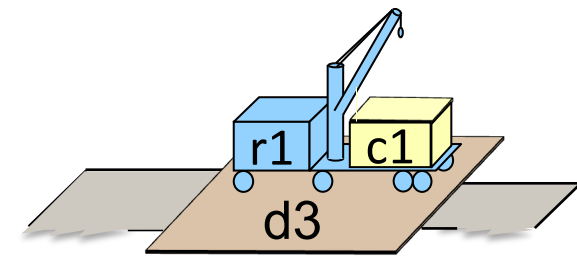
# Relevance

- Idea: when can  $a$  be useful as the last action of a plan to achieve  $g$ ?
  - ▶  $a$  makes at least one atom in  $g$  true that wasn't true already
  - ▶  $a$  doesn't make any part of  $g$  false
- $a$  is *relevant* for  $g = \{x_1=c_1, x_2=c_2, \dots, x_k=c_k\}$  if
  - ▶ at least one atom in  $g$  is also in  $\text{eff}(a)$ 
    - e.g., if  $\text{eff}(a)$  contains  $x_1 \leftarrow c_1$
  - ▶  $\text{eff}(a)$  doesn't make any atom in  $g$  false
    - e.g.,  $\text{eff}(a)$  must not contain  $x_2 \leftarrow c_2'$  (where  $c_2' \neq c_2$ )
  - ▶ whenever  $\text{pre}(a)$  requires an atom of  $g$  to be false,  $\text{eff}(a)$  makes the atom true
    - e.g., if  $\text{pre}(a)$  contains  $x_3 = c_3'$  (where  $c_3' \neq c_3$ ), then  $\text{eff}(a)$  must contain  $x_3 \leftarrow c_3$



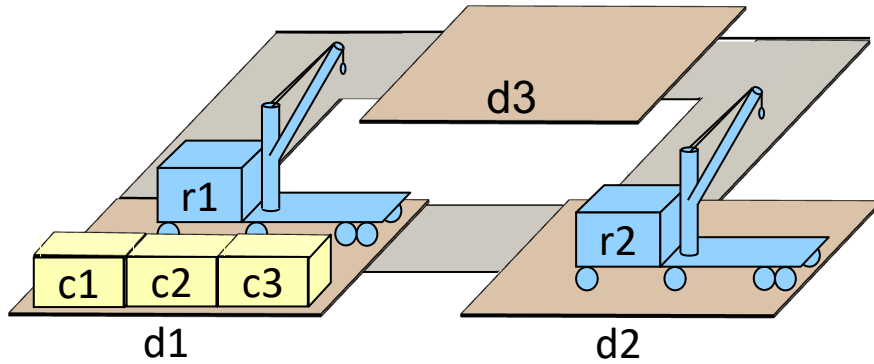
$s = \{\text{loc}(c1)=d1, \text{loc}(c2)=d1, \text{loc}(c3)=d1,$   
 $\text{loc}(r1)=d2, \text{cargo}(r1)=\text{nil},$   
 $\text{loc}(r2)=d2, \text{cargo}(r2)=\text{nil}\}$

$\text{load}(r,c,l)$   
 pre:  $\text{cargo}(r)=\text{nil}, \text{loc}(r)=l, \text{loc}(c)=l$   
 eff:  $\text{cargo}(r) \leftarrow c, \text{loc}(c) \leftarrow r$



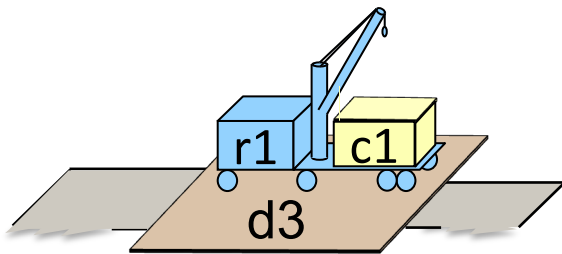
$g = \{\text{cargo}(r1)=c1, \text{loc}(r1)=d3\}$

# Relevance



$s = \{\text{loc}(c1)=d1, \text{loc}(c2)=d1, \text{loc}(c3)=d1,$   
 $\text{loc}(r1)=d2, \text{cargo}(r1)=\text{nil},$   
 $\text{loc}(r2)=d2, \text{cargo}(r2)=\text{nil}\}$

$\text{adjacent} = \{(d1,d2), (d1,d3), (d2,d1),$   
 $(d2,d3), (d3,d1), (d3,d2)\}$



$g = \{\text{cargo}(r1)=c1, \text{loc}(r1)=d3\}$

$\text{move}(r,l,m)$

pre:  $\text{loc}(r)=l, \text{adjacent}(l,m)$

eff:  $\text{loc}(r) \leftarrow m$

$\text{load}(r,c,l)$

pre:  $\text{cargo}(r)=\text{nil}, \text{loc}(r)=l, \text{loc}(c)=l$

eff:  $\text{cargo}(r) \leftarrow c, \text{loc}(c) \leftarrow r$

$\text{put}(r,l,c)$

pre:  $\text{loc}(r)=l, \text{loc}(c)=r$

eff:  $\text{cargo}(r) \leftarrow \text{nil}, \text{loc}(c) \leftarrow l$

$\text{Range}(r) = \text{Robots} = \{r1,r2\}$

$\text{Range}(l) = \text{Range}(m) = \text{Locs} = \{d1,d2,d3\}$

$\text{Range}(c) = \text{Containers} = \{c1,c2,c3\}$

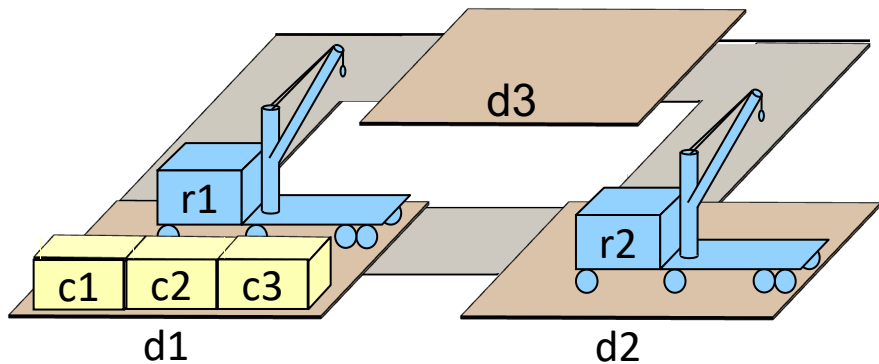
**Poll:** for each action below, is it relevant for  $g$ ?

$\text{load}(r1,c1,d1)$      $\text{load}(r1,c1,d2)$      $\text{put}(r2,c1,d3)$

$\text{move}(r1,d1,d3)$      $\text{move}(r1,d3,d1)$      $\text{move}(r1,d2,d3)$

# Inverse State Transitions

- If  $a$  is relevant for  $g$ , then  $\gamma^{-1}(g,a) = \text{pre}(a) \cup (g - \text{eff}(a))$
- If  $a$  isn't relevant for  $g$ , then  $\gamma^{-1}(g,a)$  is undefined
- Example:
  - ▶  $g = \{\text{loc}(c1)=r1\}$
  - ▶ What is  $\gamma^{-1}(g, \text{load}(r1,c1,d3))$ ?
  - ▶ What is  $\gamma^{-1}(g, \text{load}(r2,c1,d1))$ ?



$\text{move}(r,l,m)$

pre:  $\text{loc}(r)=l, \text{adjacent}(l,m)$

eff:  $\text{loc}(r) \leftarrow m$

$\text{load}(r,c,l)$

pre:  $\text{cargo}(r)=\text{nil}, \text{loc}(r)=l, \text{loc}(c)=l$

eff:  $\text{cargo}(r) \leftarrow c, \text{loc}(c) \leftarrow r$

$\text{put}(r,l,c)$

pre:  $\text{loc}(r)=l, \text{loc}(c)=r$

eff:  $\text{cargo}(r) \leftarrow \text{nil}, \text{loc}(c) \leftarrow l$

$\text{Range}(r) = \text{Robots}$

$\text{Range}(l) = \text{Range}(m) = \text{Locs}$

$\text{Range}(c) = \text{Containers}$

# Backward Search

Backward-search  $(\Sigma, s_0, g)$

(i)  $\pi \leftarrow \langle \rangle$

**while**  $s \neq g$  **do**

(ii)  $A' \leftarrow \{a \in A \mid a \text{ is relevant for } g\}$

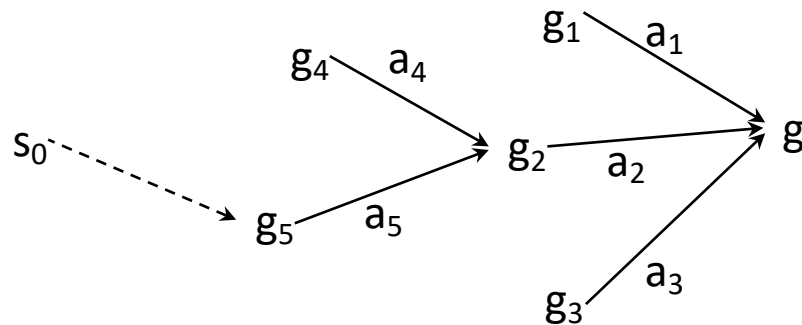
**if**  $A' = \emptyset$  **then return failure**

nondeterministically choose  $a \in A'$

(iii)  $g \leftarrow \gamma^{-1}(g, a)$

$\pi \leftarrow a \cdot \pi$

**return**  $\pi$



Cycle checking:

- After line (i), put  $Visited \leftarrow \{g_0\}$

- After line (iii), put this:

if  $g \in Visited$  then

return failure

$Visited \leftarrow Visited \cup \{g\}$

or this:

if  $\exists g' \in Visited$  s.t.  $g \Rightarrow g'$  then

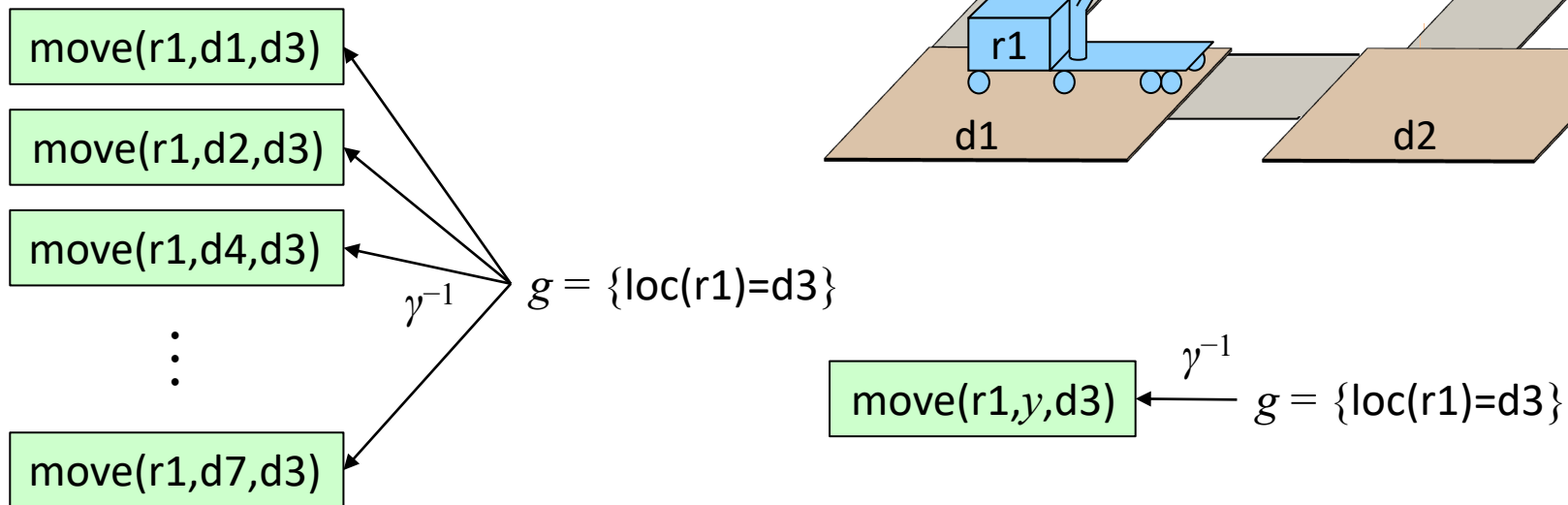
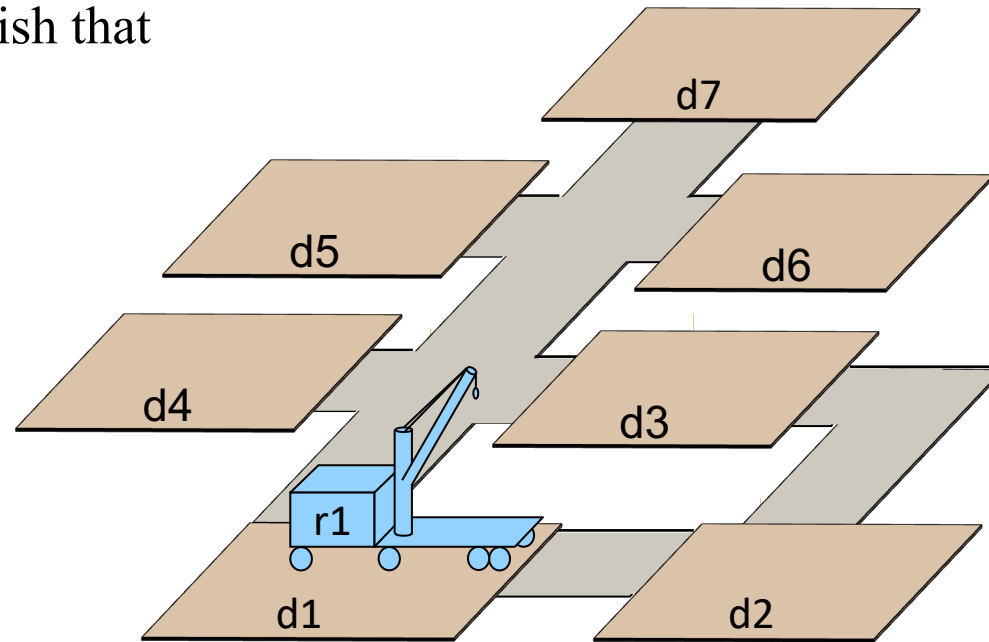
return failure

$Visited \leftarrow Visited \cup \{g\}$

- With cycle checking, sound and complete
  - ▶ If  $(\Sigma, s_0, g_0)$  is solvable, then at least one execution trace will find a solution

# Branching Factor

- Motivation for Backward-search was to reduce the branching factor
  - ▶ As written, doesn't accomplish that
- Solve this by *lifting*:
  - ▶ When possible, leave variables uninstantiated
  - ▶ Most implementations of Backward-search do this



# Lifted Backward Search

- Like Backward-search but much smaller branching factor
- Must keep track of what values were substituted for which parameters
  - ▶ I won't discuss the details
  - ▶ PSP (later) does something similar

Backward-search ( $\Sigma, s_0, g$ )

```
 $\pi \leftarrow \langle \rangle$   
while  $s \neq g$  do  
   $A' \leftarrow \{a \in A \mid a \text{ is relevant for } g\}$   
  if  $A' = \emptyset$  then return failure  
  nondeterministically choose  $a \in A'$   
   $g \leftarrow \gamma^{-1}(g, a)$   
   $\pi \leftarrow a \cdot \pi$   
return  $\pi$ 
```

For classical planning,  
this can be simplified:

Lifted-backward-search ( $\Sigma, s_0, g$ )

```
 $\pi \leftarrow$  the empty plan  
while(True) do  
  if  $s_0$  satisfies  $g$  then return  $\pi$   
   $Relevant \leftarrow \{(a, \sigma_1 \cdot \sigma_2) \mid a \text{ is an action in } \Sigma \text{ that is relevant for } g,$   
     $\sigma_1 \text{ is a substitution that standardizes } a \text{'s variables, and}$   
     $\sigma_2 \text{ is an mgu for } \sigma_1(a) \text{ and the atom of } g \text{ that } a \text{ is relevant for}\}$   
  if  $Relevant = \emptyset$  then return failure  
  nondeterministically choose a pair  $(a, \sigma) \in Relevant$   
   $\pi \leftarrow \sigma(a) \cdot \sigma(\pi)$   
   $g \leftarrow \gamma^{-1}(\sigma(g), \sigma(a))$ 
```



## 3.4. Plan-Space Planning

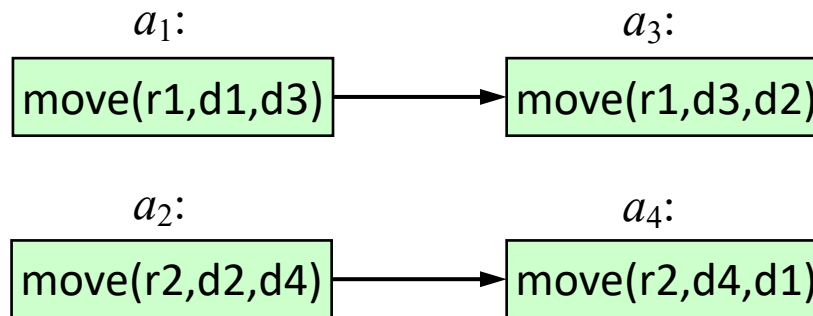
- Formulate planning as a constraint satisfaction problem
  - ▶ Use constraint-satisfaction techniques to get solutions that are more flexible than ordinary plans
    - E.g., plans in which the actions are partially ordered
    - Postpone ordering decisions until the plan is being executed
      - ▶ the actor may have a better idea about which ordering is best
- First step toward temporal planning (Chapter 18)
- Basic idea:
  - ▶ Backward search from the goal
  - ▶ Each node of the search space is a *partial plan* that contains *flaws*
    - Remove the flaws by making *refinements*
  - ▶ If successful, we'll get a *partially ordered* solution

# Definitions

- *Partially ordered plan*
  - ▶ partially ordered set of nodes
  - ▶ each node contains an action
- *Partially ordered solution* for a planning problem  $P$ 
  - ▶ partially ordered plan  $\pi$  such that every total ordering of  $\pi$  is a solution for  $P$

**Poll.** Let  $P$  be the planning problem at right, and  $\pi$  be the partially ordered plan below. Is  $\pi$  a partially ordered solution for  $P$ ?

precedence constraints:  
 $a_1 < a_3$   
 $a_2 < a_4$



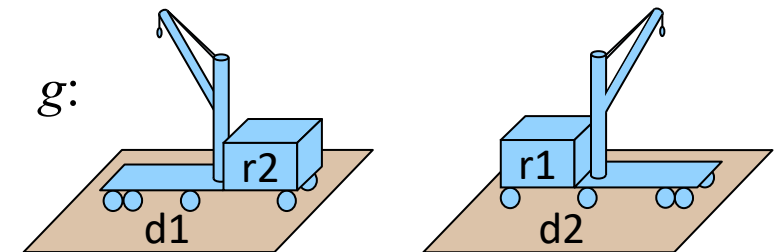
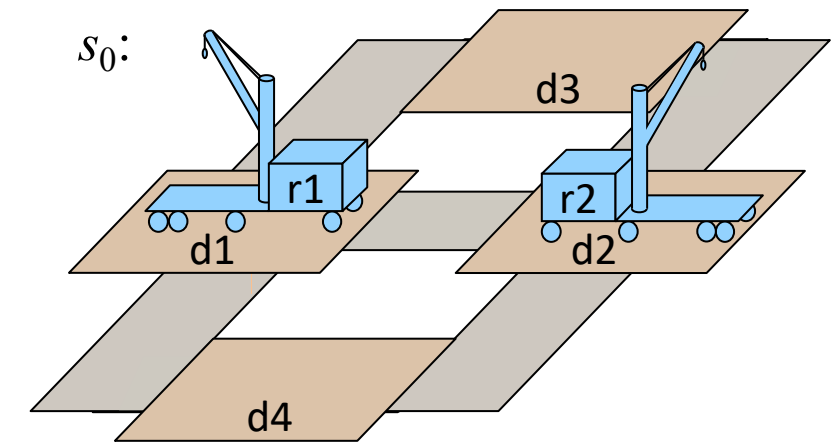
$\text{move}(r, d, d')$

pre:  $\text{loc}(r) = d, \text{occupied}(d') = \text{nil}$

eff:  $\text{loc}(r) \leftarrow d', \text{occupied}(d') \leftarrow r, \text{occupied}(d) \leftarrow \text{nil}$

$r \in \text{Robots}$

$d, d' \in \text{Docks}$



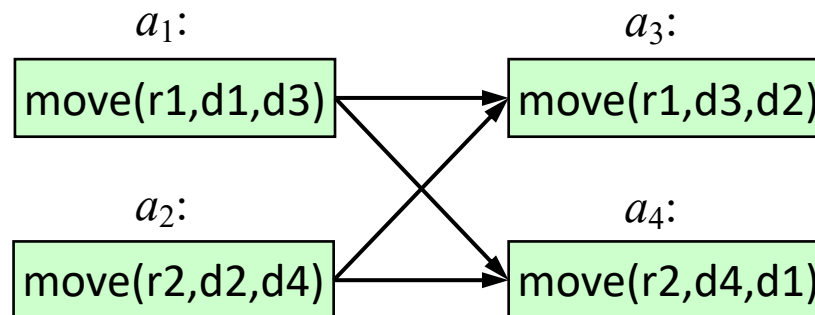
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precedence constraints:

$$a_1 < a_3, a_1 < a_4, \\ a_2 < a_3, a_2 < a_4$$



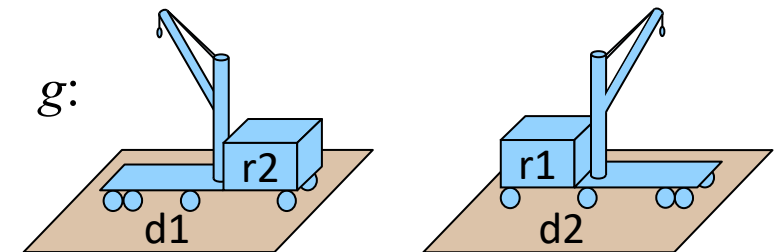
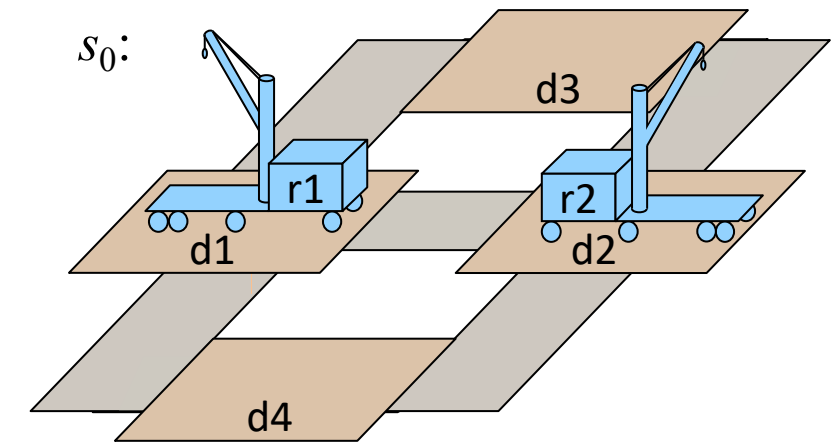
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pre:  $\text{loc}(r) = d, \text{occupied}(d') = \text{nil}$

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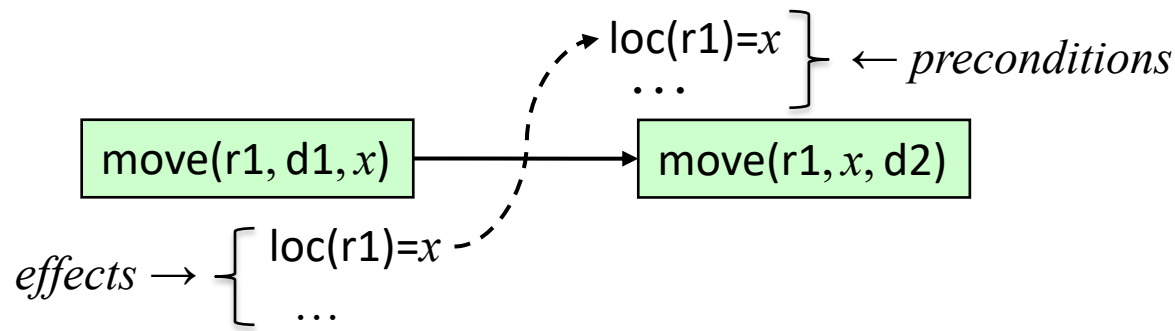
$r \in \text{Robots}$

$d, d' \in \text{Docks}$



# Definitions

- *Partial plan*
  - ▶ partially ordered set of nodes that contain *partially instantiated* actions
  - ▶ *inequality constraints*, e.g.  $z \neq x$  or  $w \neq p1$
  - ▶ *causal links* (dashed arcs)
    - constraint: action  $a$  *must* be the action that establishes action  $b$ 's precondition  $p$



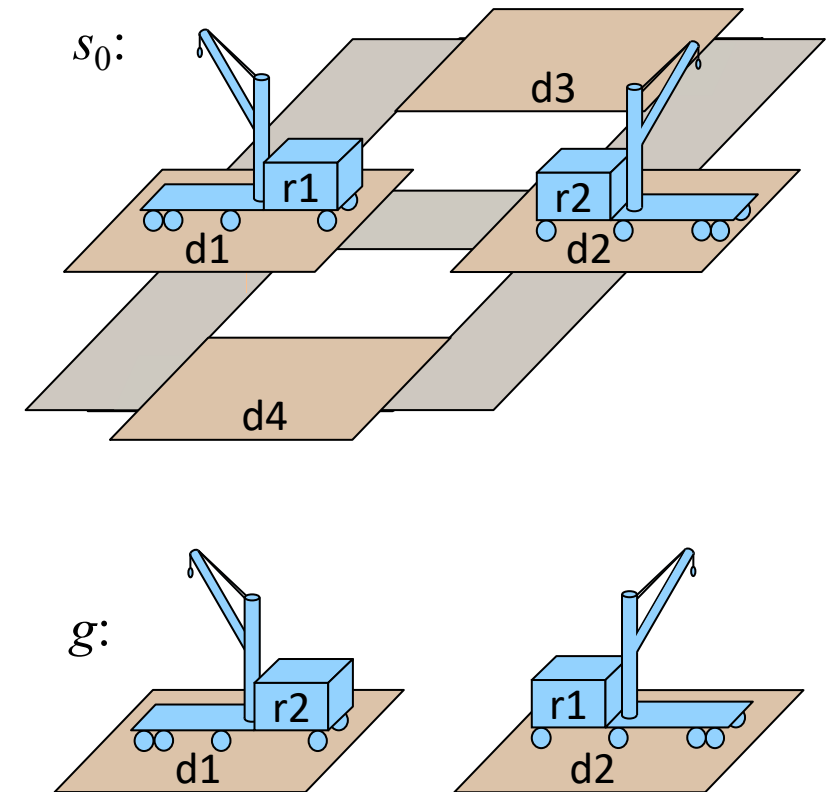
$\text{move}(r, d, d')$

pre:  $\text{loc}(r) = d, \text{occupied}(d') = \text{nil}$

eff:  $\text{loc}(r) \leftarrow d', \text{occupied}(d') \leftarrow r, \text{occupied}(d) \leftarrow \text{nil}$

$r \in \text{Robots}$

$d, d' \in \text{Docks}$



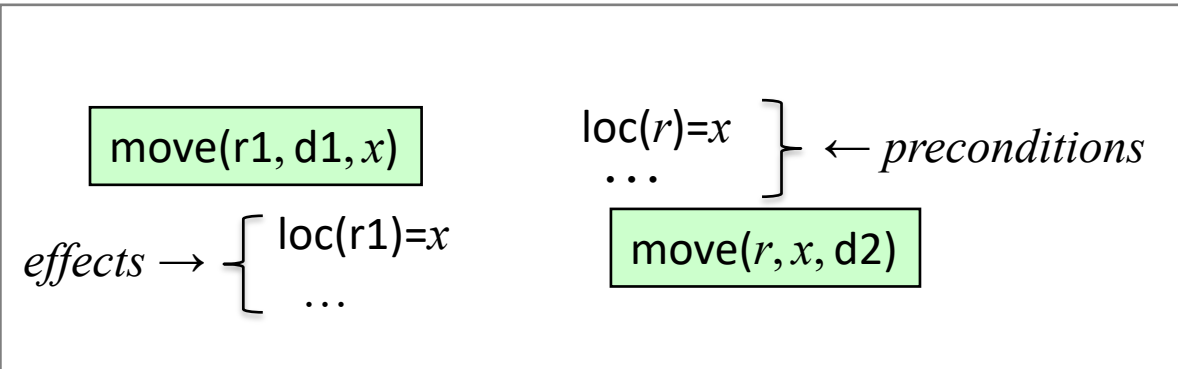
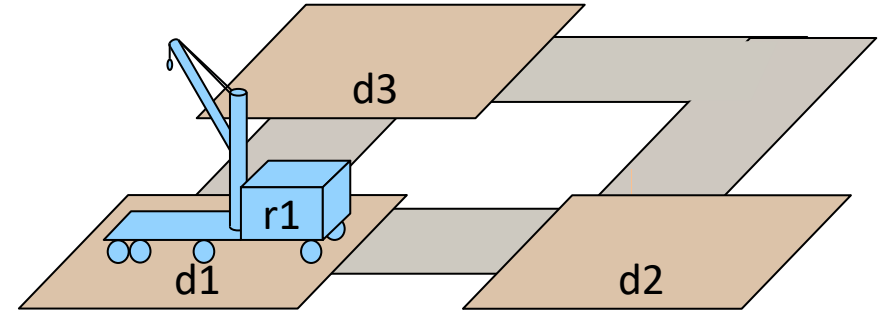
# Flaws: 1. Open Goals

- Action  $b$ , precondition  $p$ 
  - ▶  $p$  is an *open goal* if there is no causal link for  $p$
- Resolve the flaw by creating a causal link
  - ▶ Find an action  $a$  (either already in  $\pi$ , or add it to  $\pi$ ) that can *establish*  $p$ 
    - can precede  $b$
    - can have  $p$  as an effect
  - ▶ Do substitutions on variables to make  $a$  assert  $p$
  - ▶ Add an ordering constraint  $a < b$
  - ▶ Create a causal link from  $a$  to  $p$

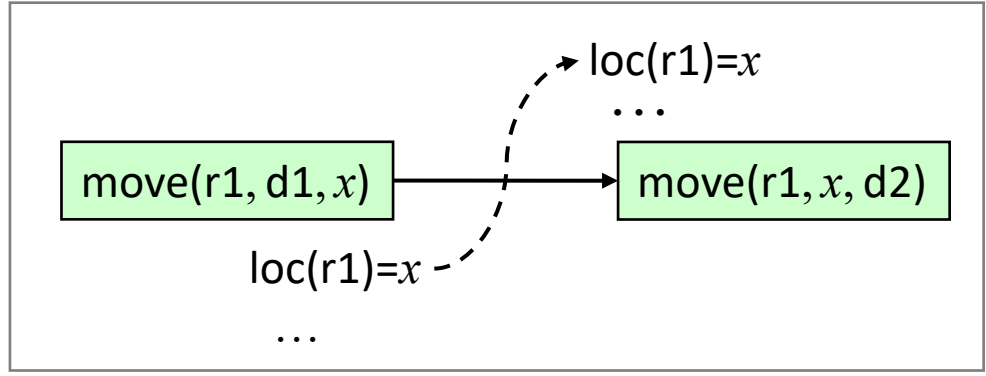
```

move( $r, d, d'$ )
pre: loc( $r$ ) =  $d$ , occupied( $d'$ ) = nil
eff: loc( $r$ )  $\leftarrow$   $d'$ , occupied( $d'$ )  $\leftarrow$   $r$ , occupied( $d$ )  $\leftarrow$  nil

 $r \in Robots$ 
 $d, d' \in Docks$ 
  
```



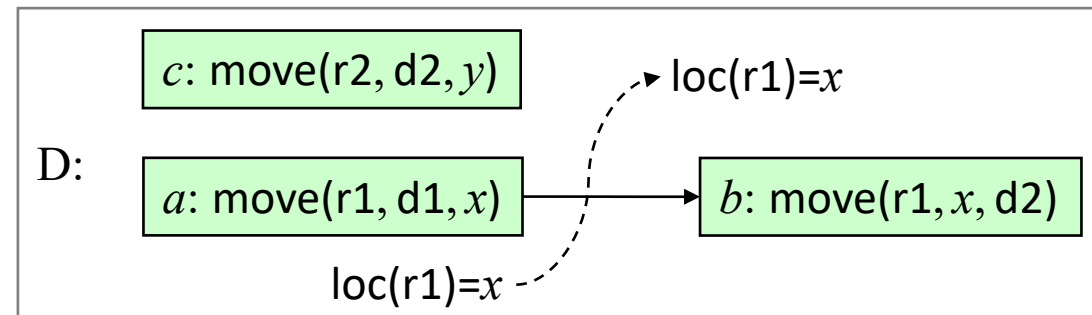
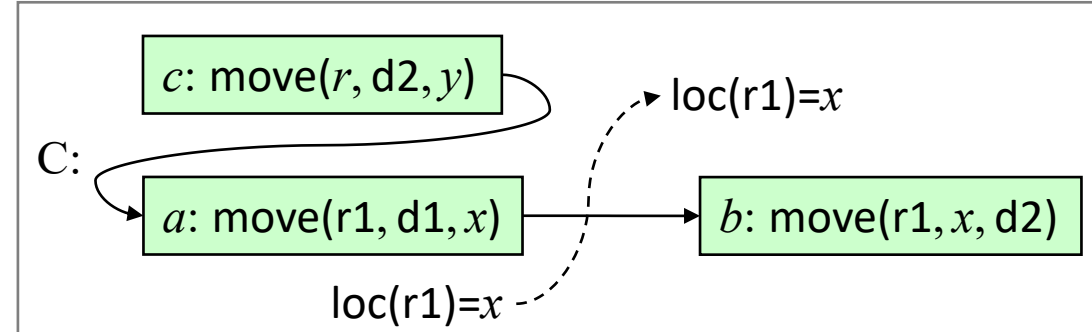
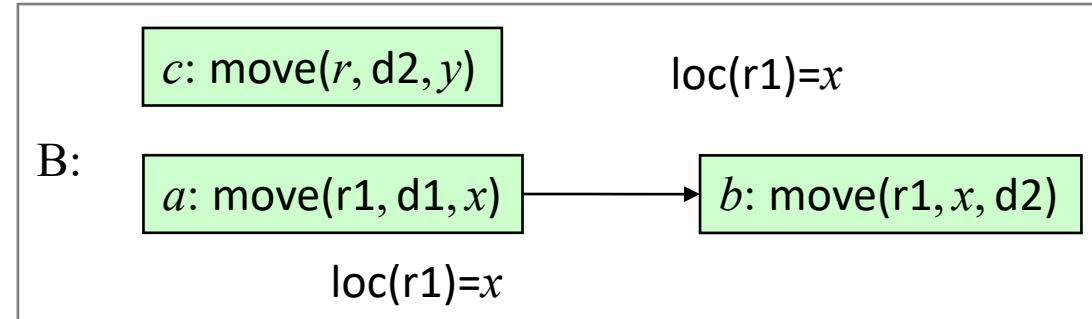
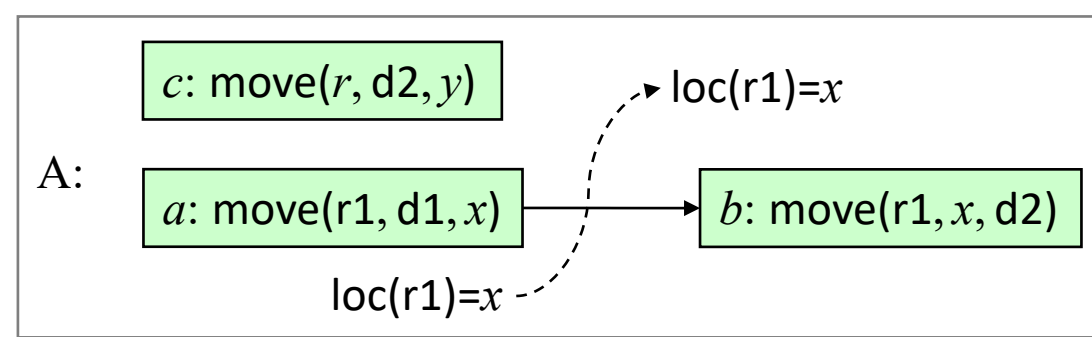
substitute  
 $r \leftarrow r1$



## Flaws: 2. Threats

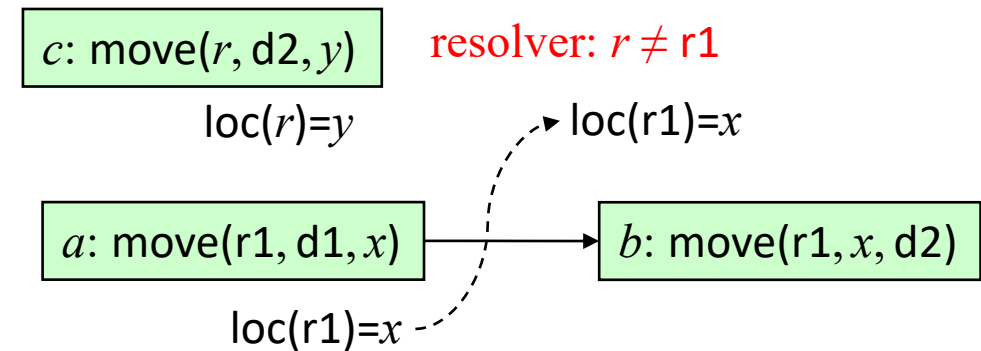
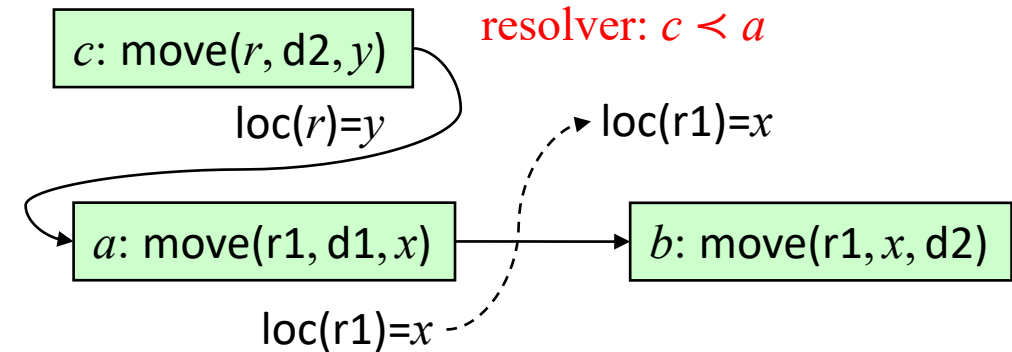
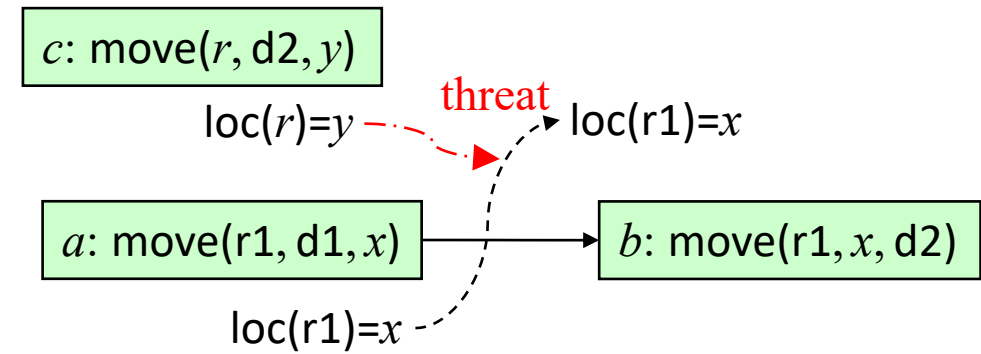
- Let  $l$  be a causal link from an effect of action  $a$  to a precondition  $p$  of action  $b$
- Action  $c$  *threatens*  $l$  if  $c$  may come between  $a$  and  $b$  and  $c$  may affect  $p$ 
  - “ $c$  may come between  $a$  and  $b$ ” means the plan’s current ordering constraints don’t prevent it
    - plan doesn’t already have  $c < a$  or  $b < c$
  - “ $c$  may affect  $p$ ” means
    - can substitute values for variables such that  $c$ ’s effects either make  $p$  true or make  $p$  false

**Poll.** In each of the cases at right, does action  $c$  threaten the causal link?



# Resolving Threats

- Suppose action  $c$  threatens a causal link  $l$  from an effect of action  $a$  to a precondition  $p$  of action  $b$
- Three possible resolvers:
  1. Add a precedence constraint  $c < a$
  2. Add a precedence constraint  $b < c$
  3. Add inequality constraints that prevent  $c$  from affecting  $p$
- Each of these is applicable iff it doesn't make the plan inconsistent
  - ▶ e.g., 2 isn't applicable if the plan already has  $c < b$

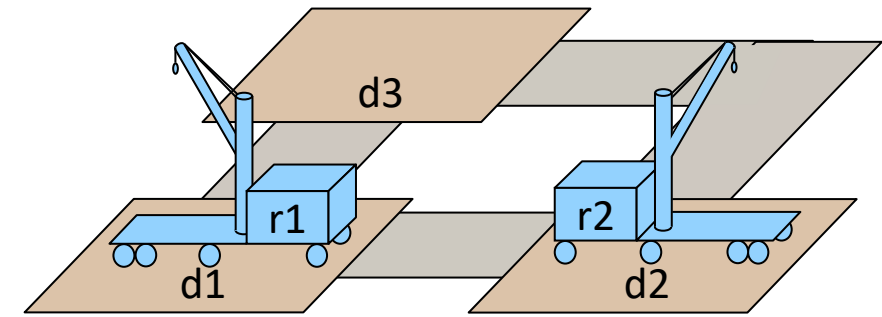


# PSP Algorithm

PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

- (i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return** failure
  - (ii) nondeterministically choose  $\rho \in R$   
modify  $\pi$  by applying  $\rho$  to it
- return**  $\pi$



- 2 open goals
- no threats



$move(r, d, d')$

pre:  $loc(r) = d, occupied(d') = nil$

eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$



# PSP Algorithm

PSP ( $\Sigma, \pi$ )

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$R \leftarrow \{\text{all feasible resolvers for } f\}$

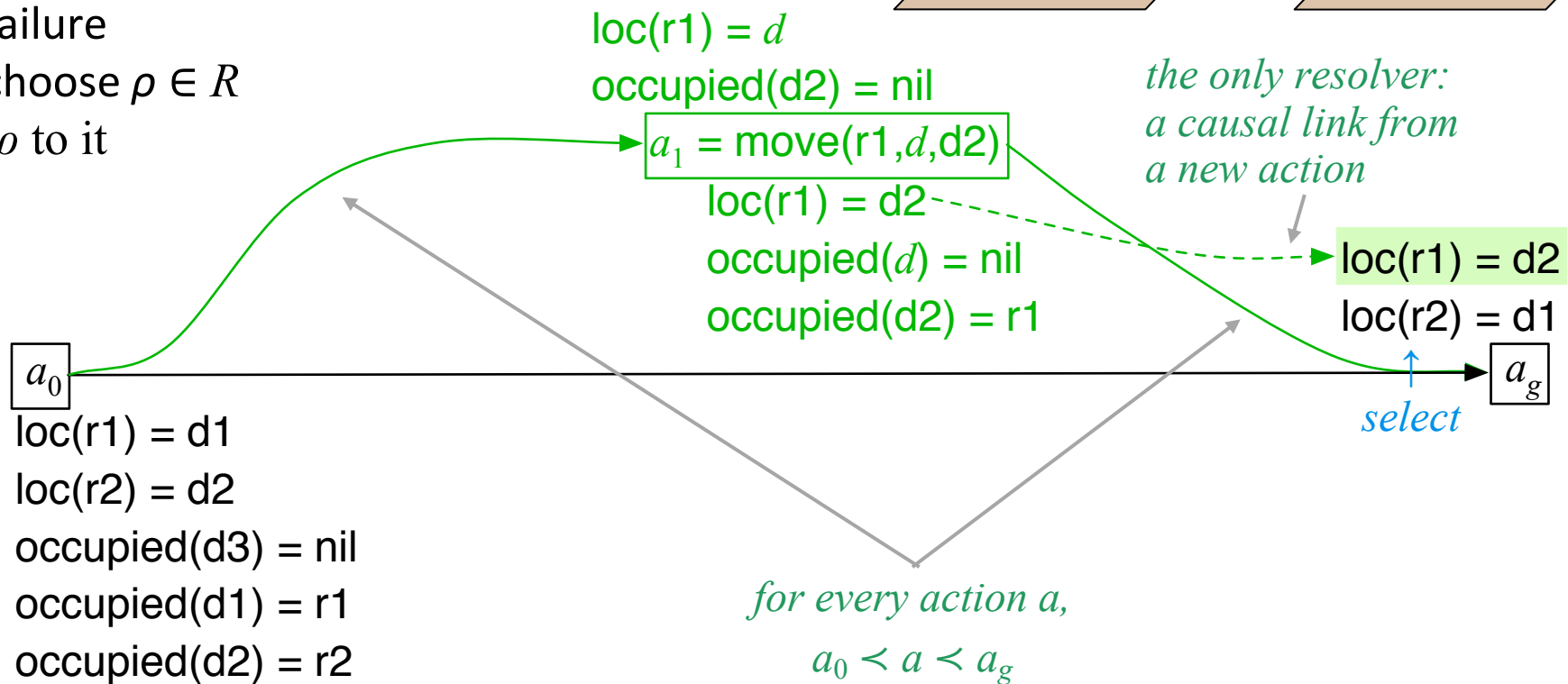
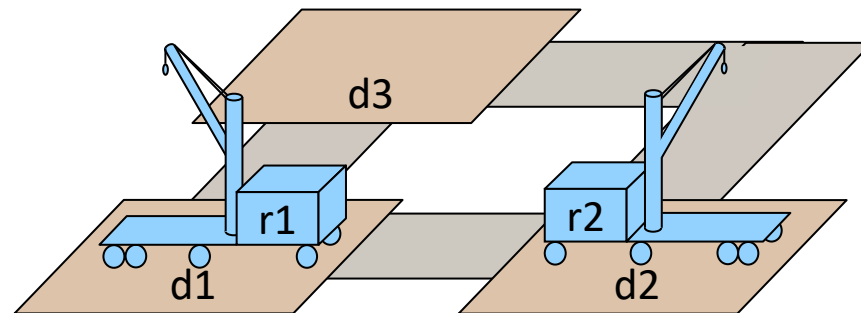
**if**  $R = \emptyset$  **then return failure**

(ii) nondeterministically choose  $\rho \in R$

modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 3 open goals
- no threats



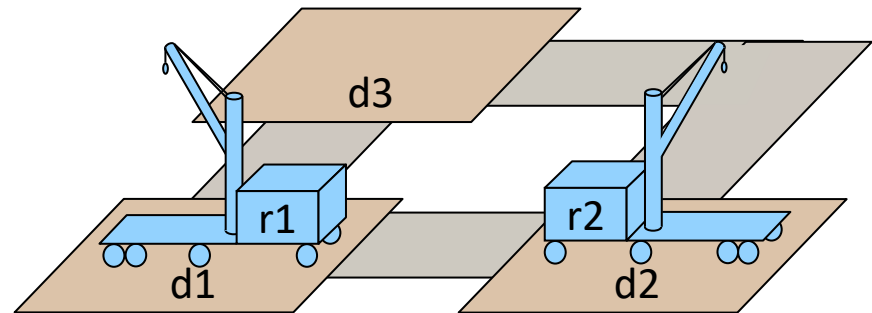
$\text{move}(r, d, d')$

pre:  $\text{loc}(r) = d, \text{occupied}(d') = \text{nil}$

eff:  $\text{loc}(r) \leftarrow d', \text{occupied}(d') = r, \text{occupied}(d) = \text{nil}$

**Poll.** Above, I said “the only resolver”. Is that correct?

# PSP Algorithm



PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

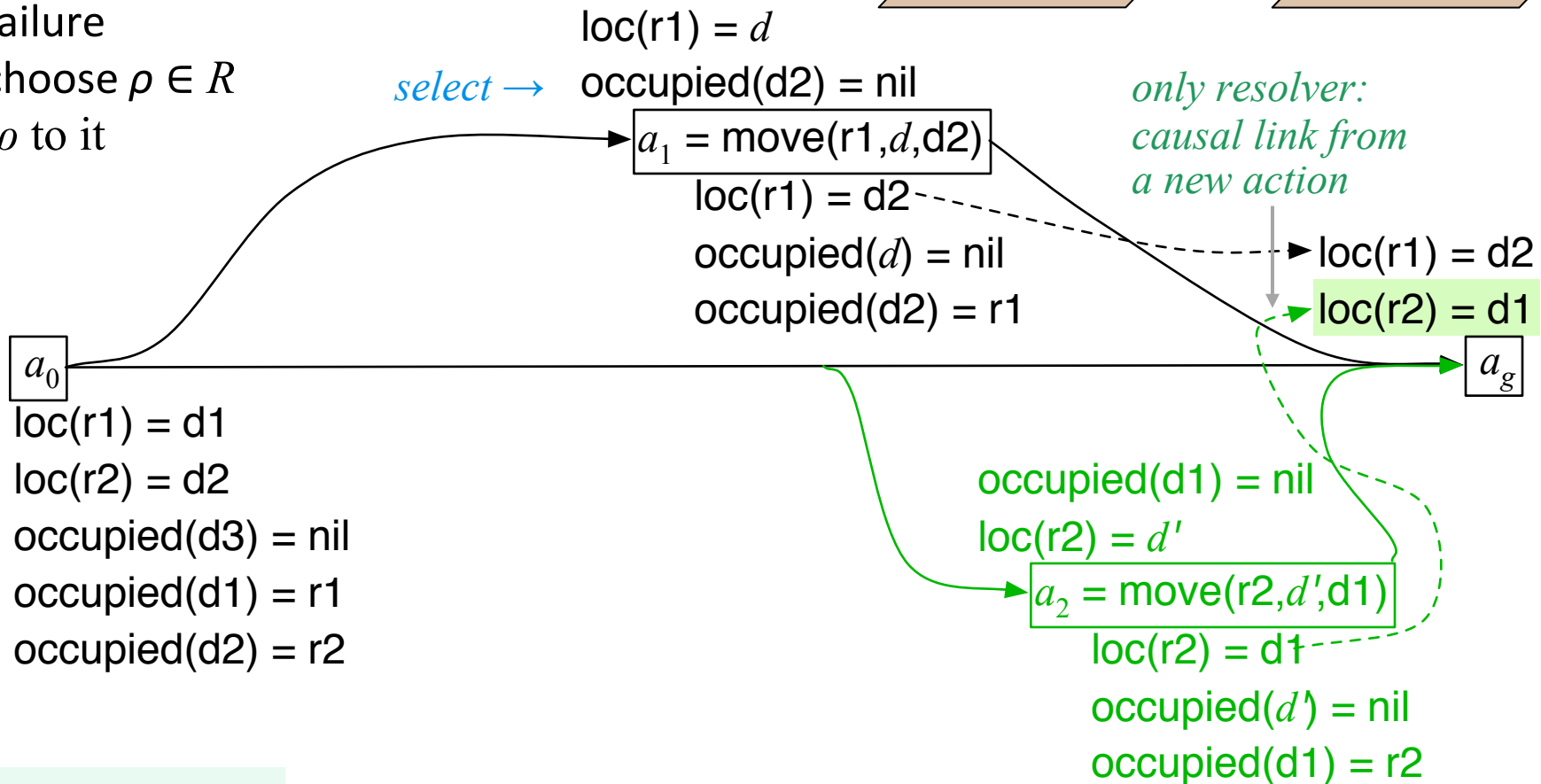
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**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 4 open goals
- no threats



$\text{move}(r, d, d')$

pre: loc( $r$ ) =  $d$ , occupied( $d'$ ) = nil

eff: loc( $r$ )  $\leftarrow$   $d'$ , occupied( $d'$ ) =  $r$ , occupied( $d$ ) = nil

PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$

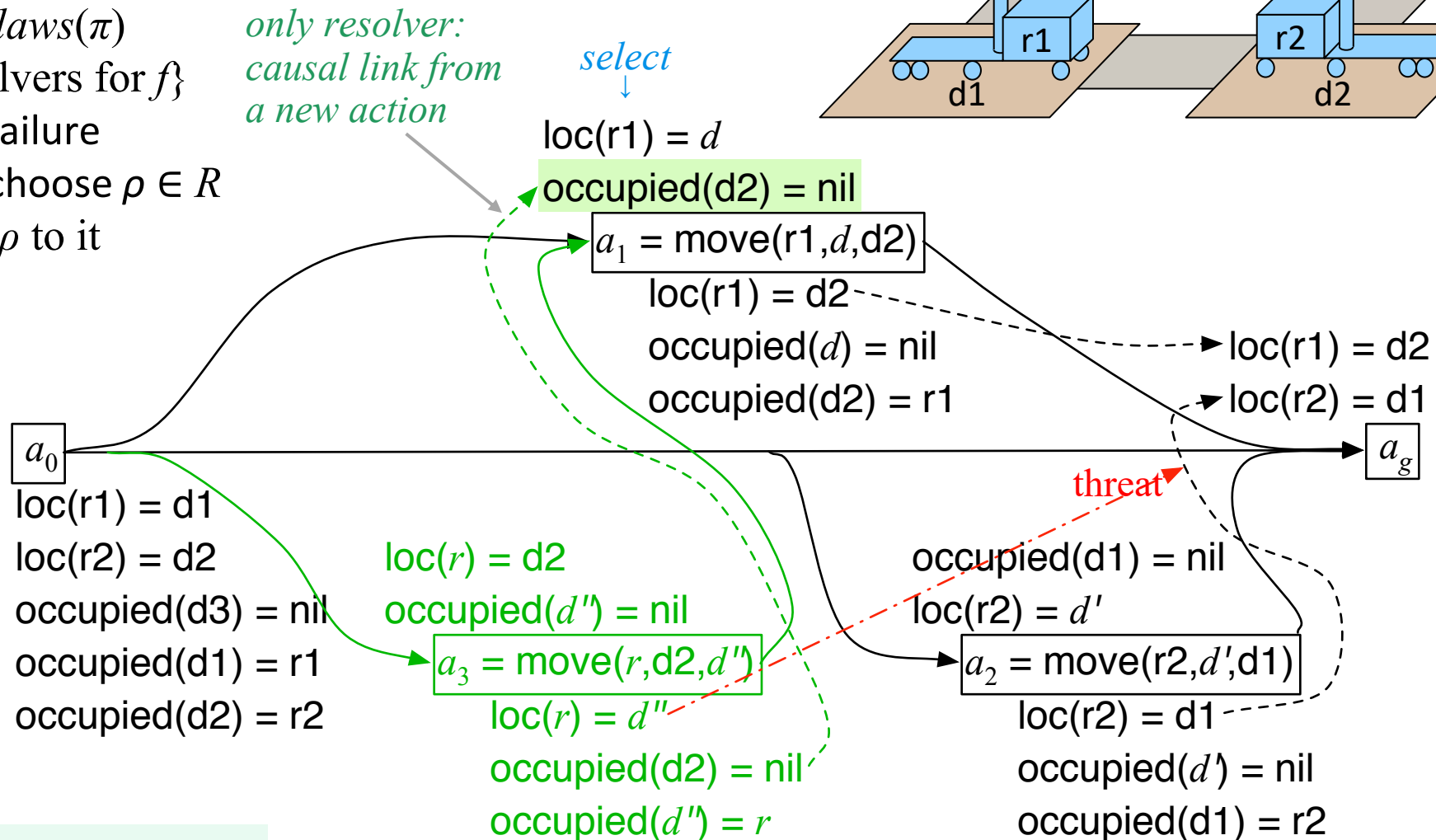
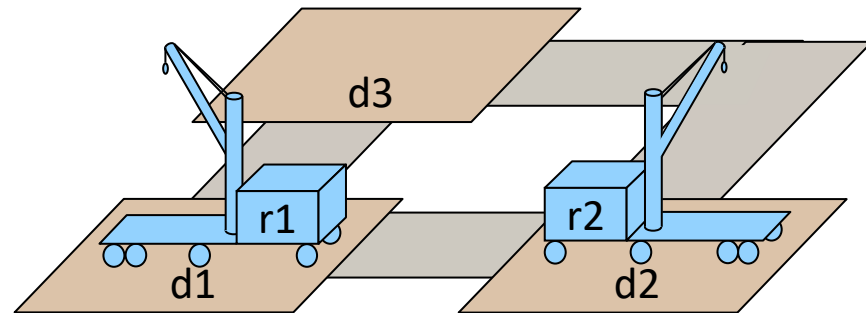
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(ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 5 open goals
- 1 threat

# PSP Algorithm



*only resolver:  
causal link from  
a new action*

*select*

$loc(r1) = d$

$occupied(d2) = nil$

$a_1 = move(r1, d, d2)$

$loc(r1) = d2$

$occupied(d) = nil$

$occupied(d2) = r1$

$loc(r1) = d2$

$loc(r2) = d1$

$a_0$

$loc(r1) = d1$

$loc(r2) = d2$

$occupied(d3) = nil$

$occupied(d1) = r1$

$occupied(d2) = r2$

$loc(r) = d2$

$occupied(d'') = nil$

$a_3 = move(r, d2, d'')$

$loc(r) = d''$

$occupied(d2) = nil$

$occupied(d'') = r$

$occupied(d1) = nil$

$loc(r2) = d'$

$a_2 = move(r2, d', d1)$

$loc(r2) = d1$

$occupied(d') = nil$

$occupied(d1) = r2$

$a_g$

*threat*

**Poll:** does  $a_3$  threaten  $a_1$ 's precondition  $loc(r1)=d$ ?

PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$

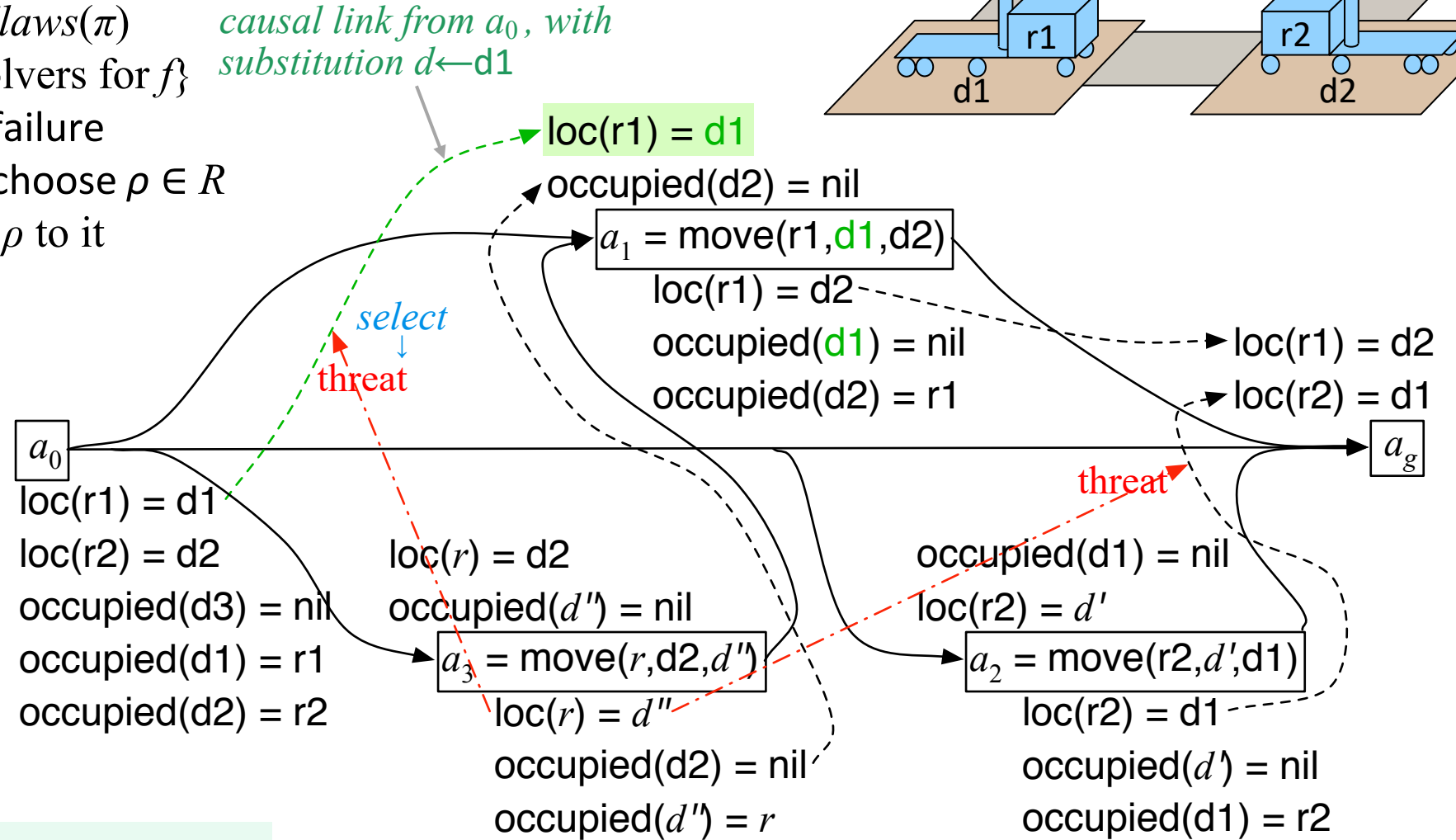
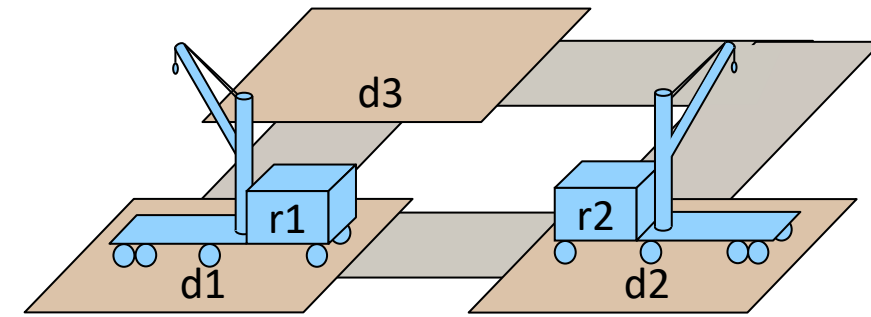
**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 4 open goals
- 2 threats

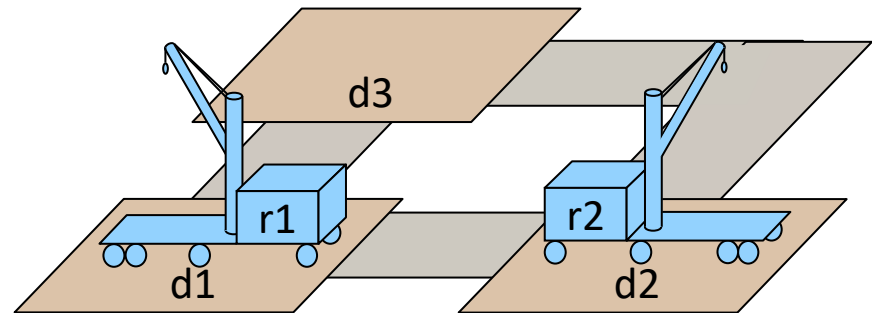
# PSP Algorithm



$\text{move}(r, d, d')$   
 pre:  $loc(r) = d, occupied(d') = nil$   
 eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

**Poll:** does  $a_3$  threaten the causal link for  $a_g$ 's precondition  $loc(r1)=d2$ ?

# PSP Algorithm



PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

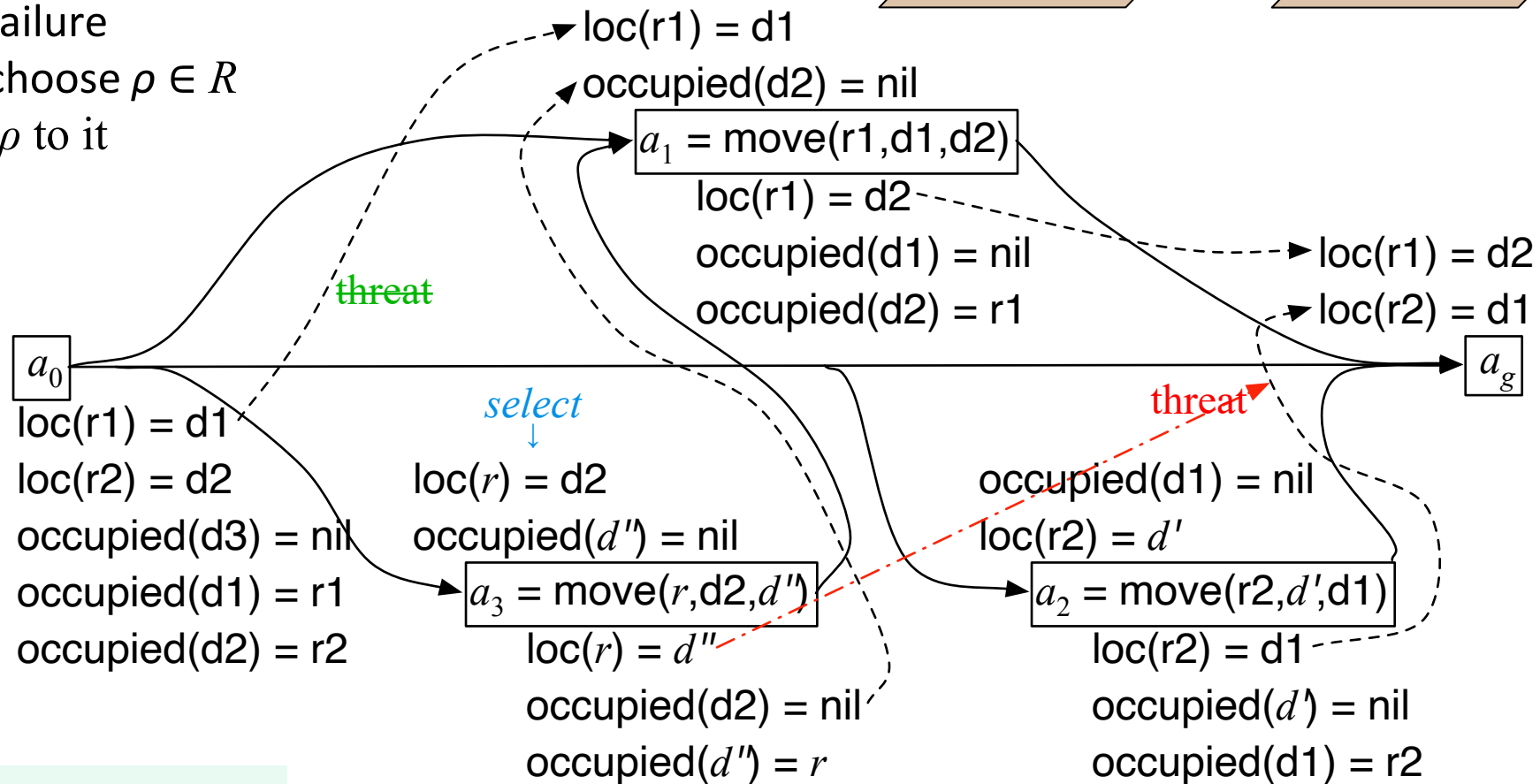
(i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 4 open goals
- 1 threat

Constraint:  $r \neq r1$

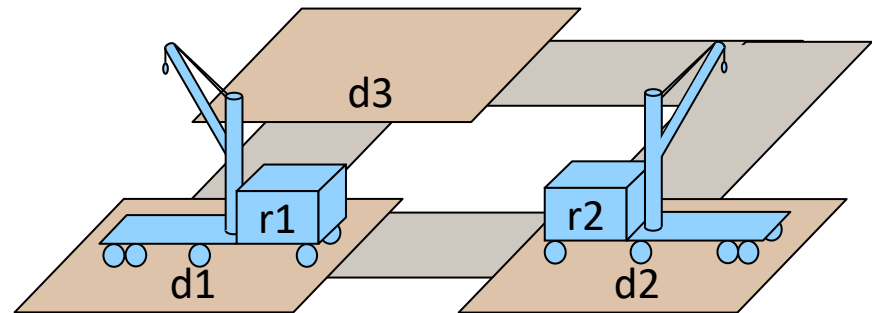


$\text{move}(r, d, d')$

pre: loc( $r$ ) =  $d$ , occupied( $d'$ ) = nil

eff: loc( $r$ )  $\leftarrow d'$ , occupied( $d'$ ) =  $r$ , occupied( $d$ ) = nil

# PSP Algorithm



PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$

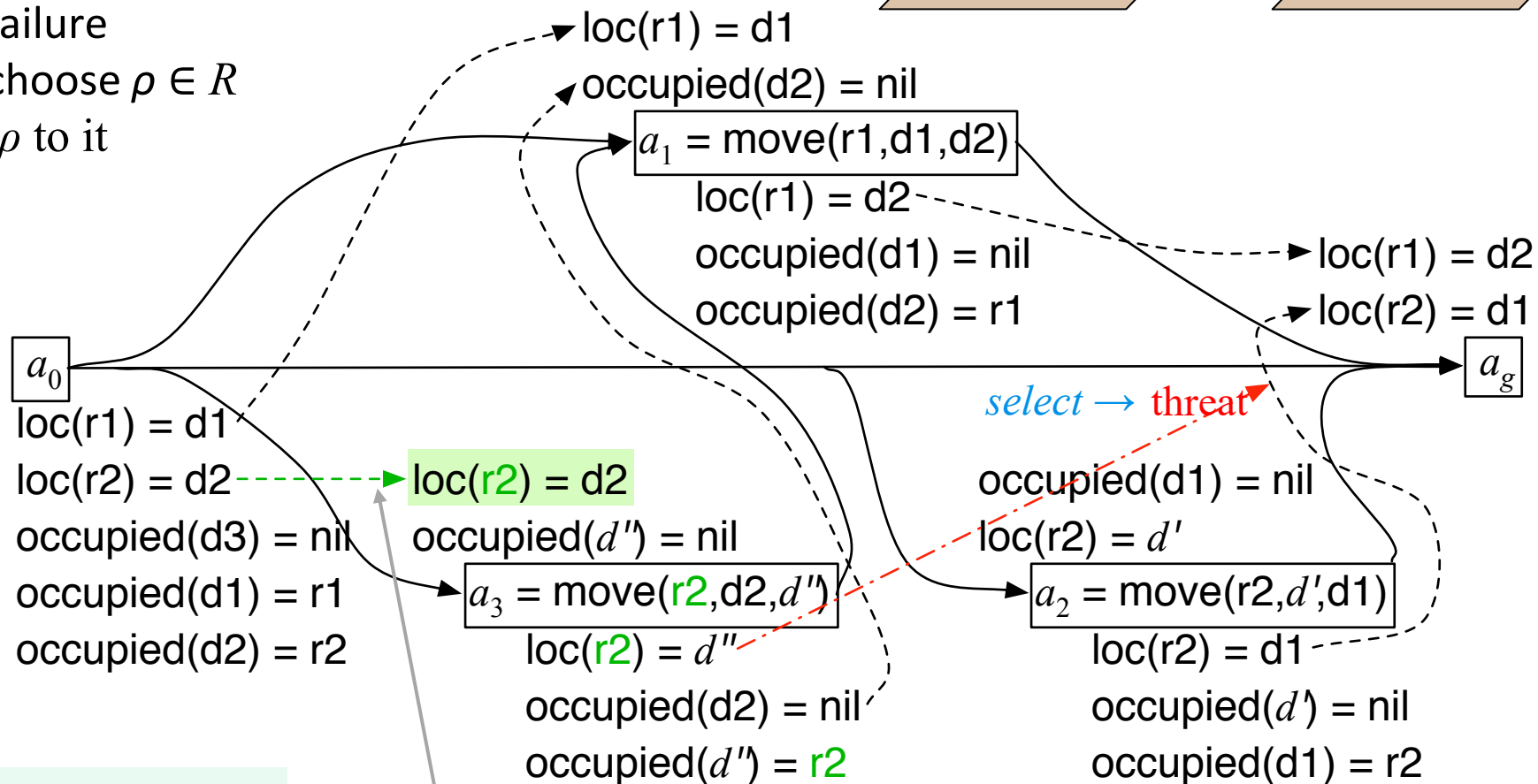
**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 3 open goals
- 1 threat

**Constraint:**  $r \neq r1$



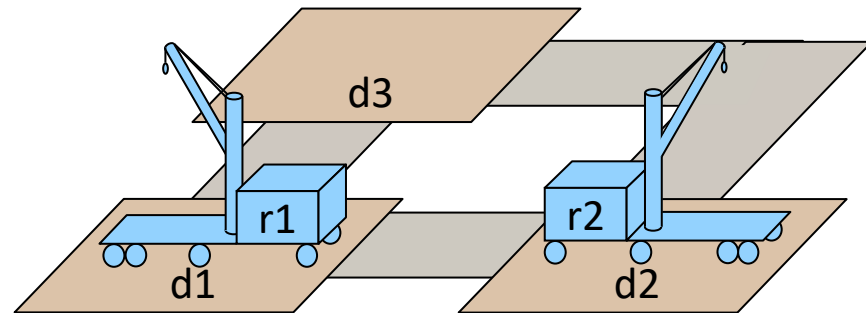
*causal link from  $a_0$   
 with substitution  $r \leftarrow r2$*

$move(r, d, d')$

pre:  $loc(r) = d, occupied(d') = nil$

eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

# PSP Algorithm



PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$

$R \leftarrow \{\text{all feasible resolvers for } f\}$

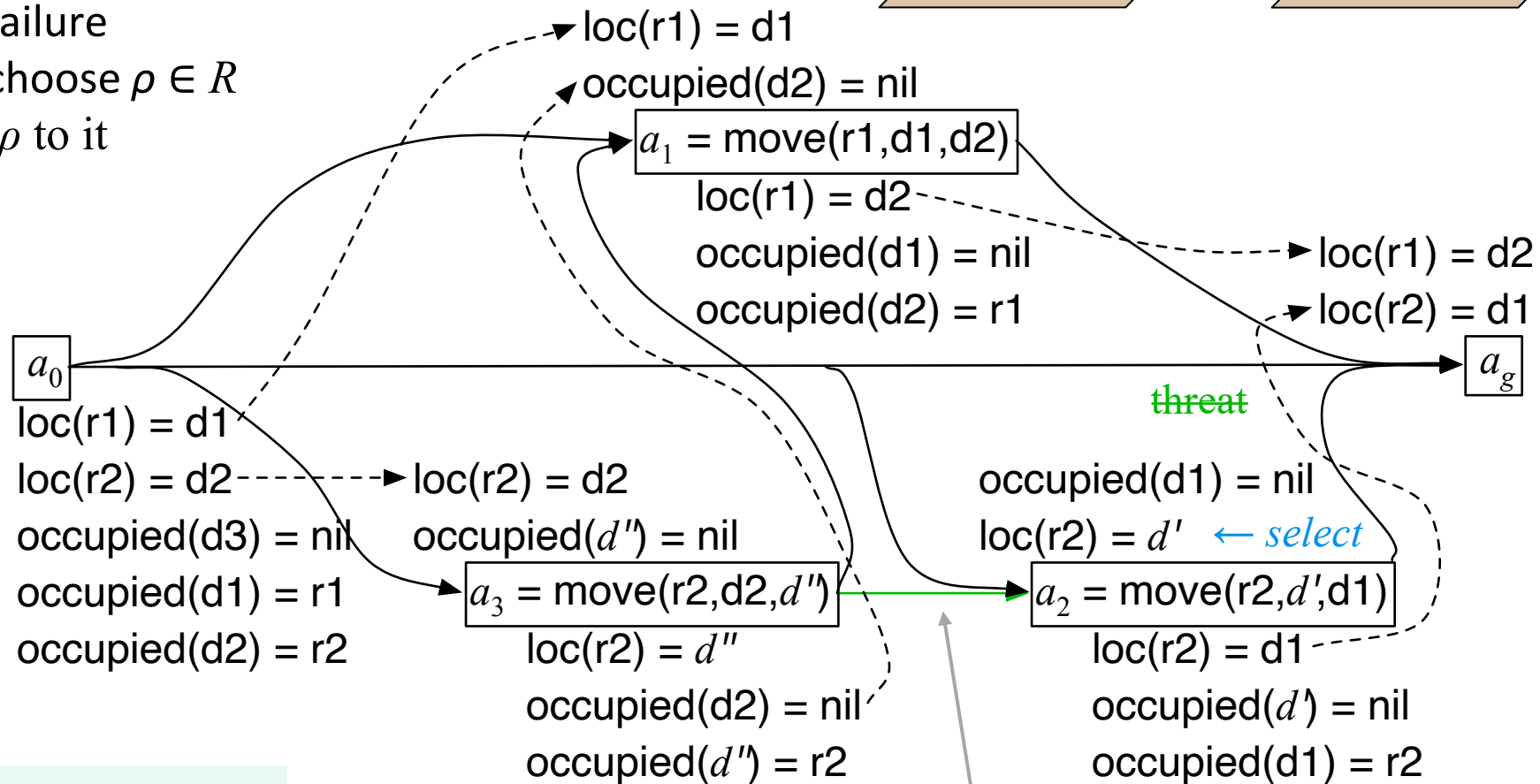
**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$

modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 3 open goals
- no threats

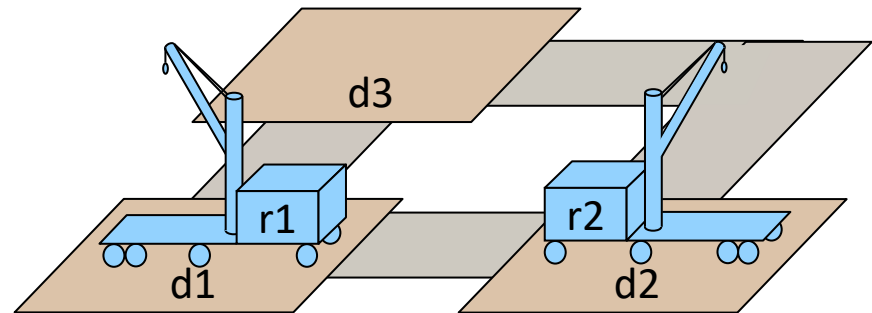


$move(r, d, d')$

pre:  $loc(r) = d, occupied(d') = nil$

eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

# PSP Algorithm



PSP ( $\Sigma, \pi$ )

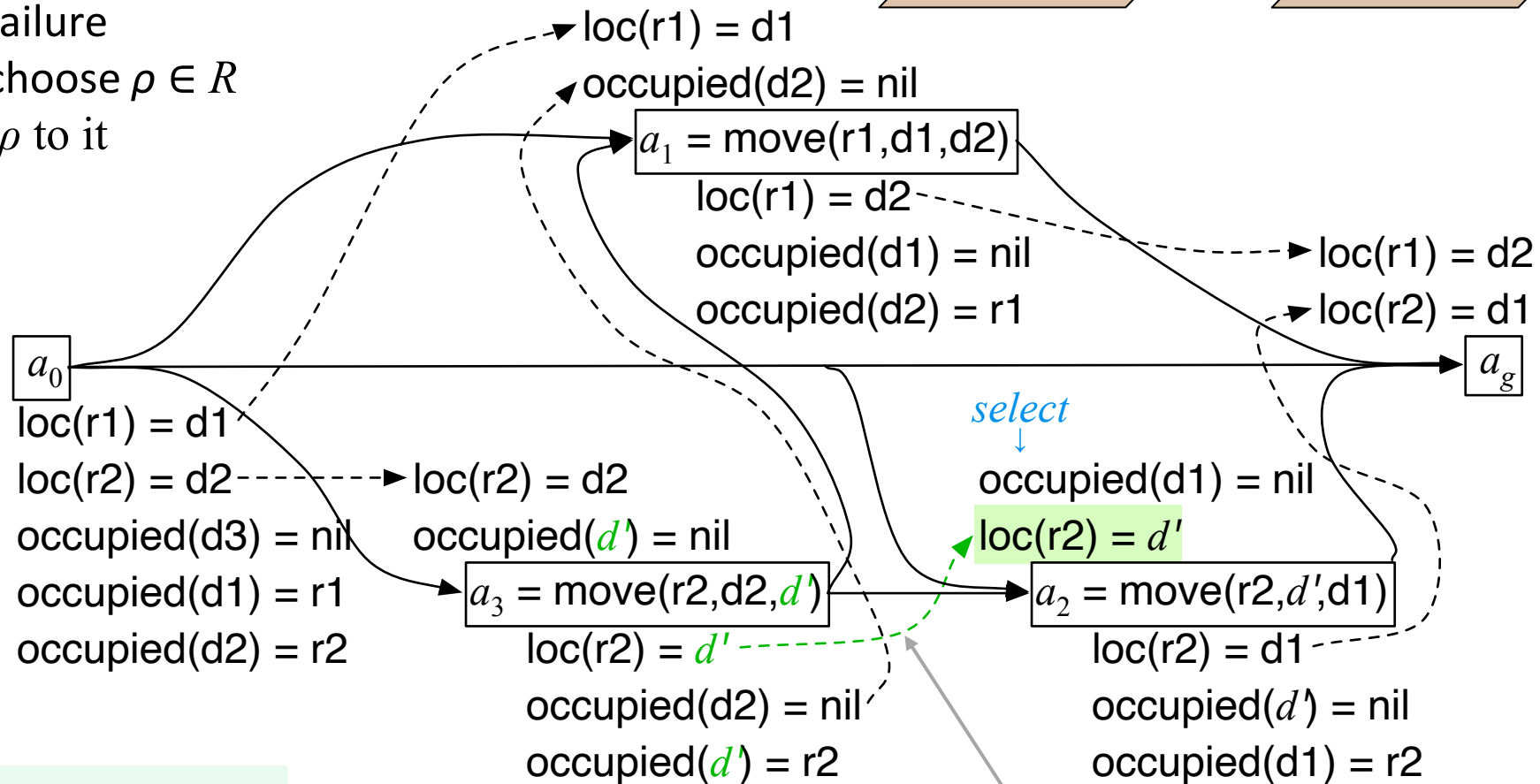
**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 2 open goals
- no threats

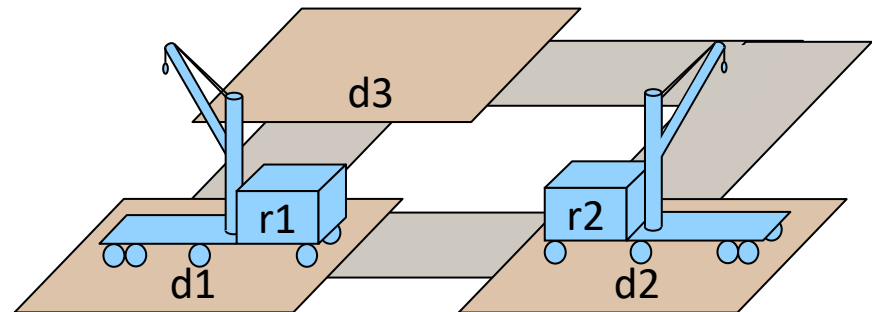


$move(r, d, d')$   
 pre:  $loc(r) = d, occupied(d') = nil$   
 eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

*causal link from  $a_3$   
 with substitution  $d'' \leftarrow d'$*



# PSP Algorithm



PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$

$R \leftarrow \{\text{all feasible resolvers for } f\}$

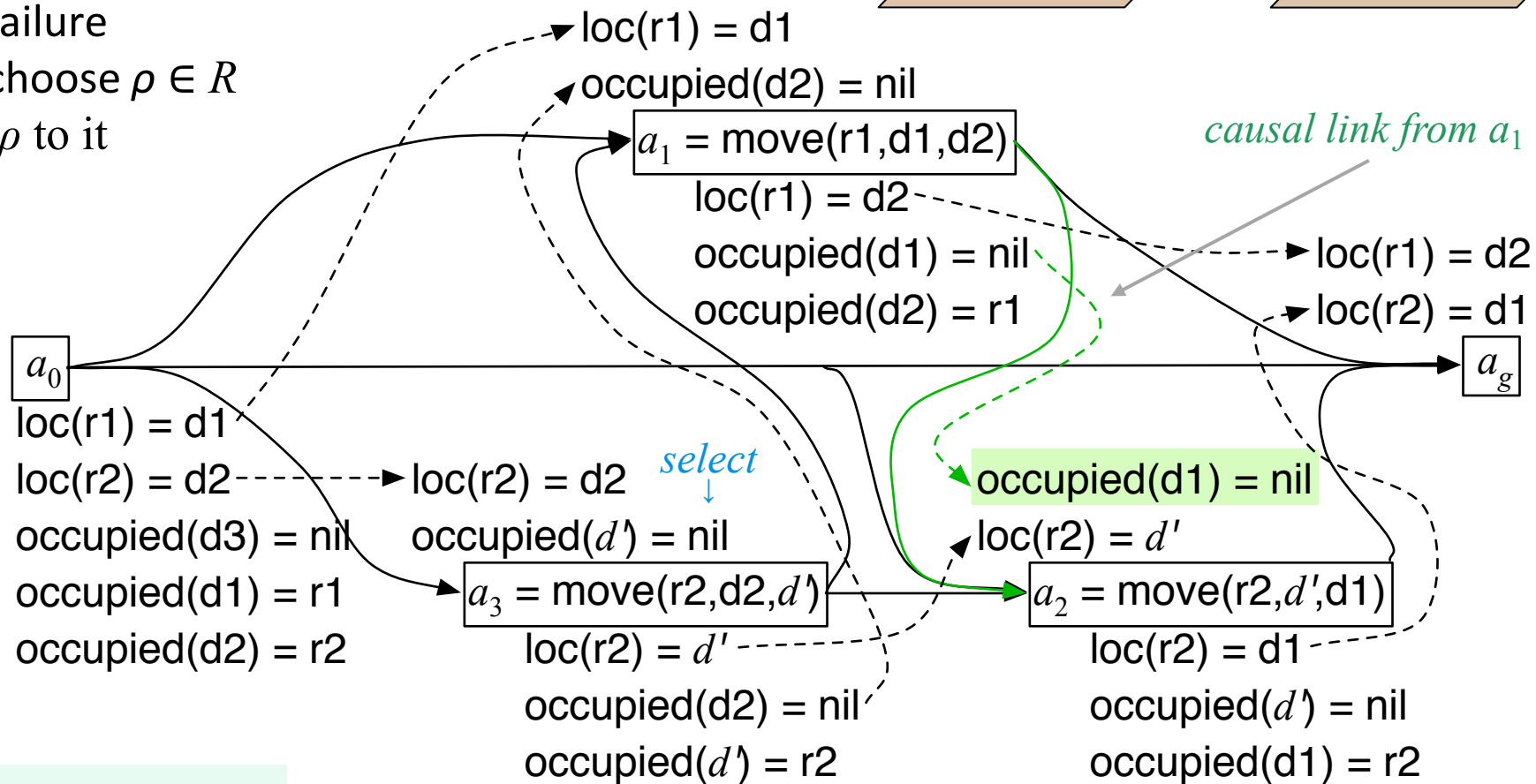
**if**  $R = \emptyset$  **then return failure**

(ii) nondeterministically choose  $\rho \in R$

modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- 1 open goal
- no threats



$\text{move}(r, d, d')$

pre:  $loc(r) = d, occupied(d') = nil$

eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

PSP ( $\Sigma, \pi$ )

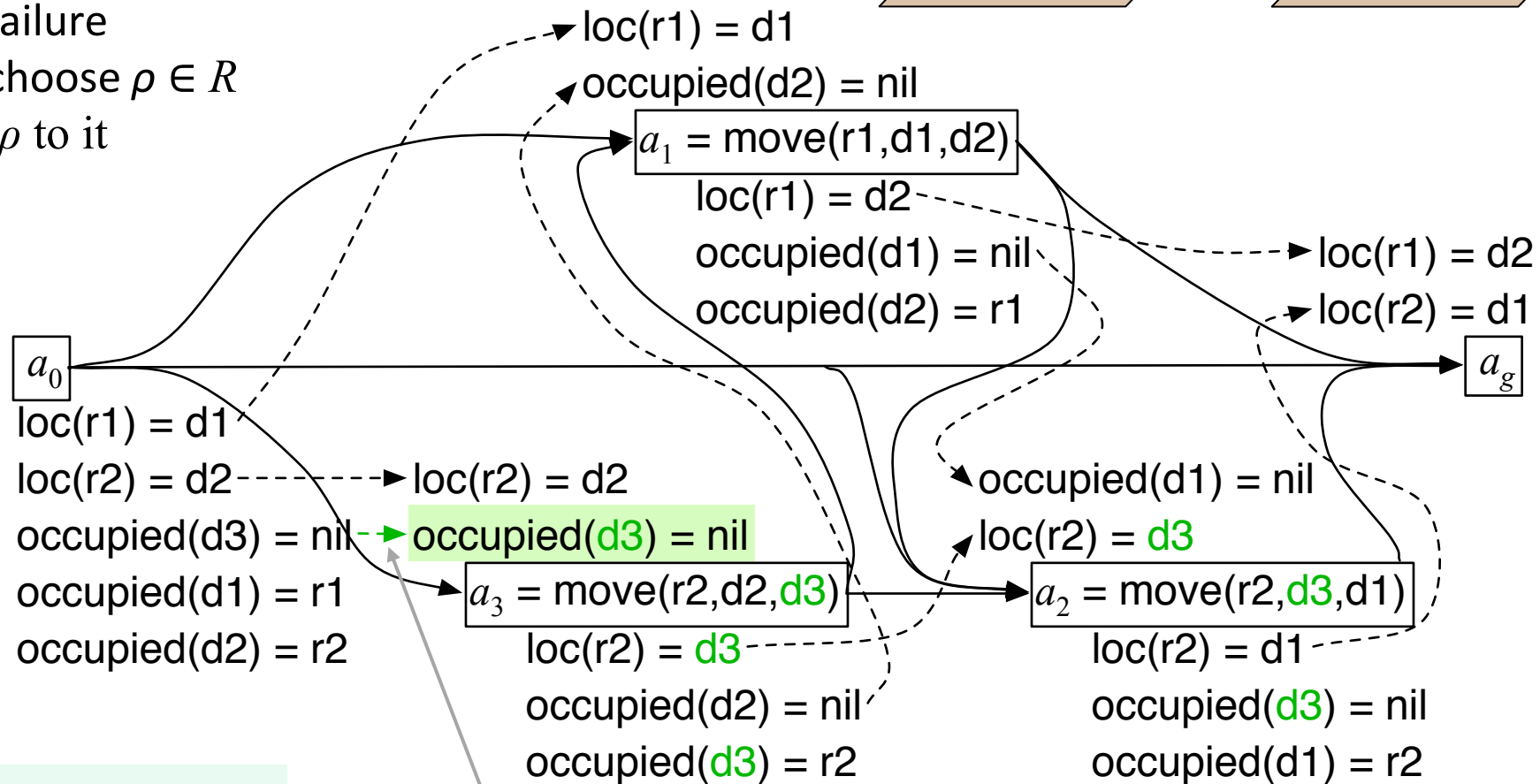
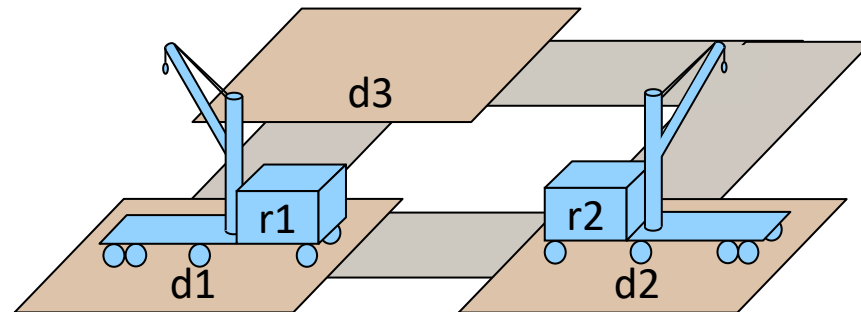
**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it  
**return**  $\pi$

- no open goals
- no threats
- we're done

# PSP Algorithm



*causal link from  $a_0$   
 with substitution  $d' \leftarrow d3$*

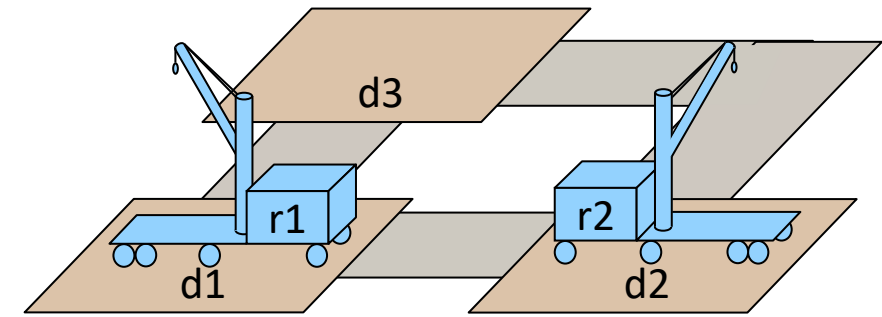
$move(r, d, d')$   
 pre:  $loc(r) = d, occupied(d') = nil$   
 eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

# PSP Algorithm

PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

- (i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return** failure
  - (ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it
- return**  $\pi$



- The solution we found:  $a_0 \rightarrow \text{move}(r2, d2, d3) \rightarrow \text{move}(r1, d1, d2) \rightarrow \text{move}(r2, d3, d1) \rightarrow a_g$
- Another:  $a_0 \rightarrow \text{move}(r2, d2, d3) \rightarrow \text{move}(r1, d1, d2) \rightarrow \text{move}(r2, d3, d1) \rightarrow \text{move}(r1, d2, d3) \rightarrow \text{move}(r2, d1, d2) \rightarrow \text{move}(r1, d3, d1) \rightarrow a_g$
- Infinitely many others  $a_0 \rightarrow \text{move}(r2, d2, d3) \rightarrow \text{move}(r1, d1, d2) \rightarrow \text{move}(r2, d3, d1) \rightarrow a_g$

$\text{move}(r, d, d')$

pre:  $\text{loc}(r) = d, \text{occupied}(d') = \text{nil}$

eff:  $\text{loc}(r) \leftarrow d', \text{occupied}(d') = r, \text{occupied}(d) = \text{nil}$

# PSP Algorithm

PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$

$R \leftarrow \{\text{all feasible resolvers for } f\}$

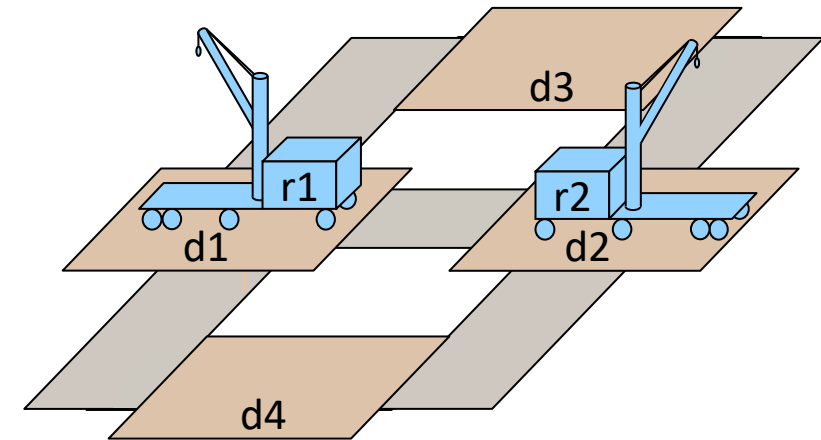
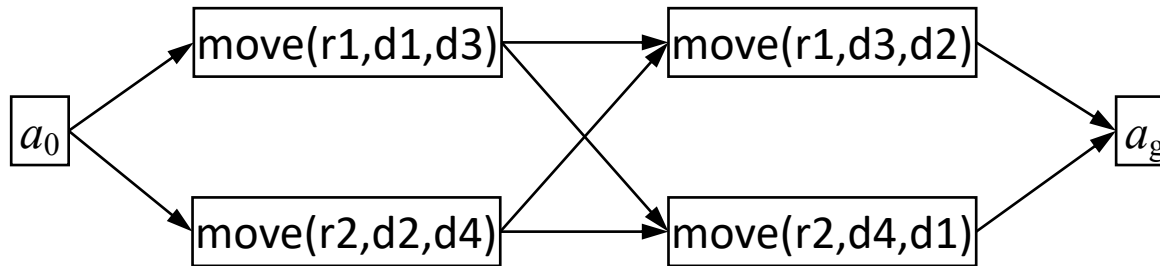
**if**  $R = \emptyset$  **then return** failure

(ii) nondeterministically choose  $\rho \in R$

modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

- Add another location to the planning domain
- Still have all of the solutions on the previous page
- There also are partially-ordered solutions



$move(r, d, d')$

pre:  $loc(r) = d, occupied(d') = nil$

eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

PSP ( $\Sigma, \pi$ )

**while**  $Flaws(\pi) \neq \emptyset$  **do**

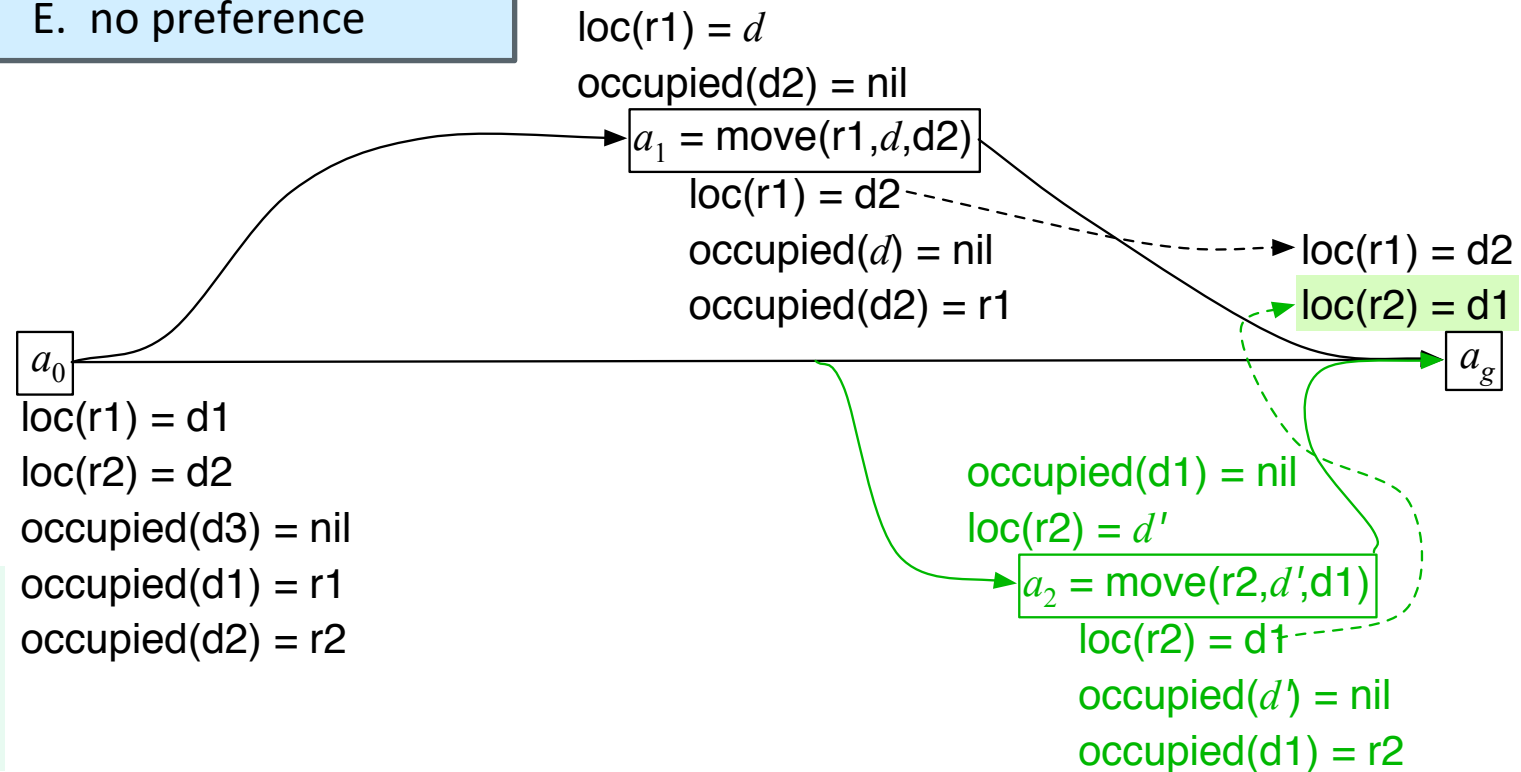
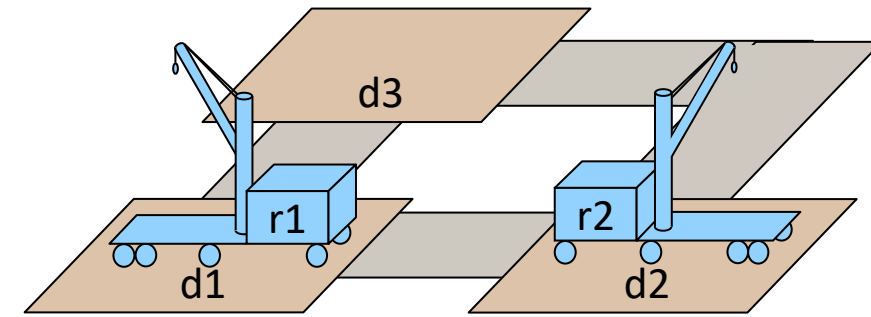
- (i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return failure**
  - (ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it
- return**  $\pi$

- Selecting a flaw to resolve in PSP  $\approx$  selecting a variable to instantiate in a CSP
  - ▶ AND-branch in both cases
- *Fewest Alternatives First (FAF)*:
  - ▶ select flaw with fewest resolvers $\approx$  Minimum Remaining Values (MRV) heuristic for CSPs

## Selecting a Flaw

**Poll:** which flaw would FAF select first?

- A.  $loc(r1)=d$
- B.  $occupied(d2) = nil$
- C.  $occupied(d1) = nil$
- D.  $loc(r2)=d'$
- E. no preference



$\text{move}(r, d, d')$   
 pre:  $loc(r) = d, occupied(d') = nil$   
 eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

PSP  $(\Sigma, \pi)$

# Choosing a Resolver

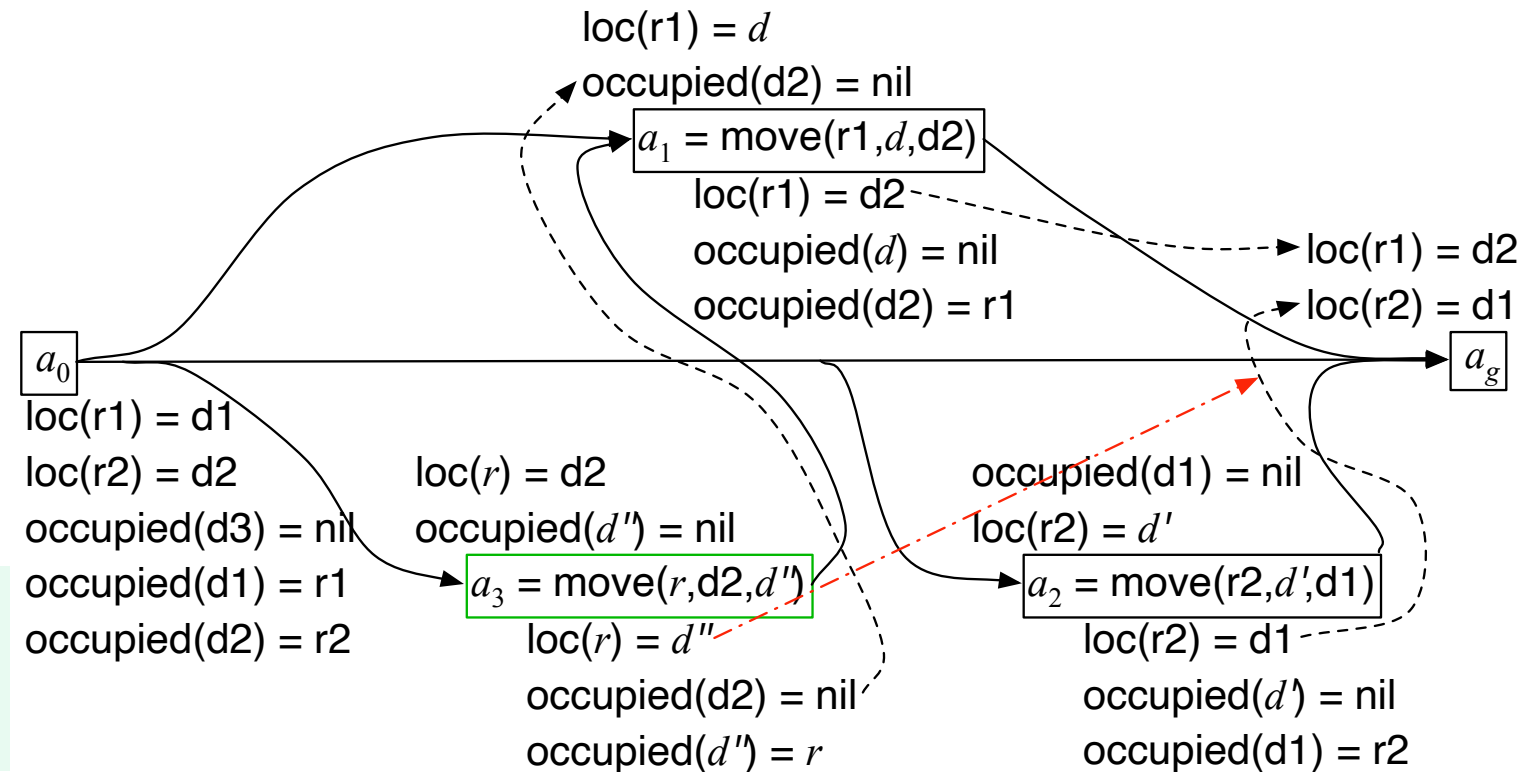
**while**  $Flaws(\pi) \neq \emptyset$  **do**

- (i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return failure**
  - (ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it
- return**  $\pi$

**Poll:** for  $loc(r1)=d$ , which resolver would LCR choose first?

- A. causal link from a new action
- B. causal link from  $a_0$ , with  $d \leftarrow d1$
- C. causal link from  $a_3$ , with  $r \leftarrow r1, d'' \leftarrow d$
- D. no preference

- Choosing a resolver for a flaw  $\approx$  assigning a value to a variable in a CSP
  - ▶ In both cases, an OR-branch
- *Least Constraining Resolver (LCR):*
  - ▶ prefer resolver that rules out the fewest resolvers for the other flaws $\approx$  Least Constraining Value (LCV) heuristic for CSPs



$\text{move}(r, d, d')$

pre:  $loc(r) = d, occupied(d') = nil$

eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

PSP  $(\Sigma, \pi)$

# Choosing a Resolver

**while**  $Flaws(\pi) \neq \emptyset$  **do**

(i) arbitrarily select  $f \in Flaws(\pi)$

$R \leftarrow \{ \text{all feasible resolvers for } f \}$

**if**  $R = \emptyset$  **then return failure**

(ii) nondeterministically choose  $\rho \in R$

modify  $\pi$  by applying  $\rho$  to it

**return**  $\pi$

• ~~Least Constraining Resolver (LCR):~~

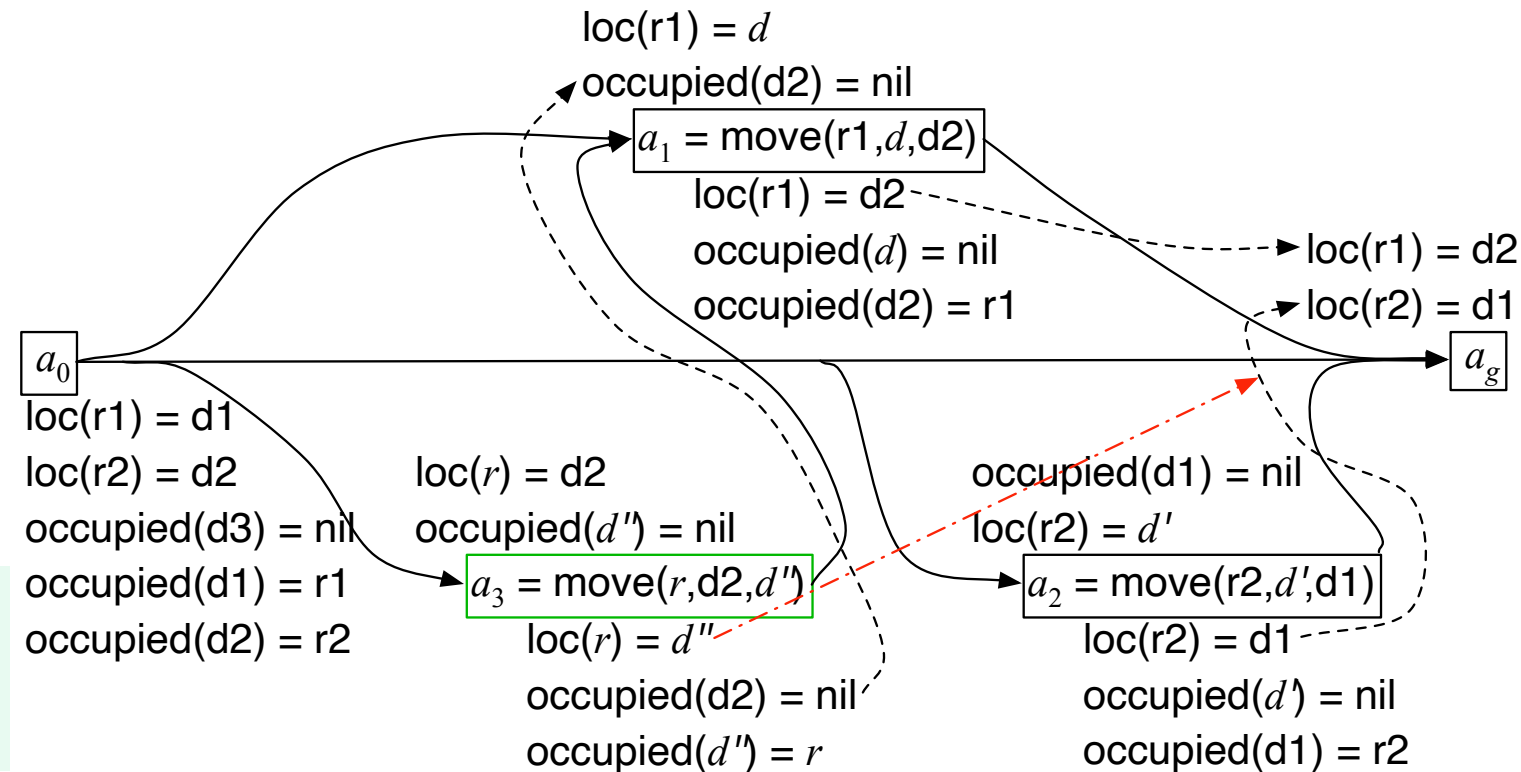
- ▶ prefer resolver that rules out the fewest resolvers for the other flaws
- $\approx$  Least Constraining Value (LCV)
- heuristic for CSPs

• Problem (in PSP but not in CSPs):

- ▶ LCR can keep adding new actions forever

**Poll:** for  $loc(r1)=d$ , which resolver would LCR choose first?

- A. causal link from a new action
- B. causal link from  $a_0$ , with  $d \leftarrow d1$
- C. causal link from  $a_3$ , with  $r \leftarrow r1, d'' \leftarrow d$
- D. no preference



$move(r, d, d')$

pre:  $loc(r) = d, occupied(d') = nil$

eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

PSP ( $\Sigma, \pi$ )

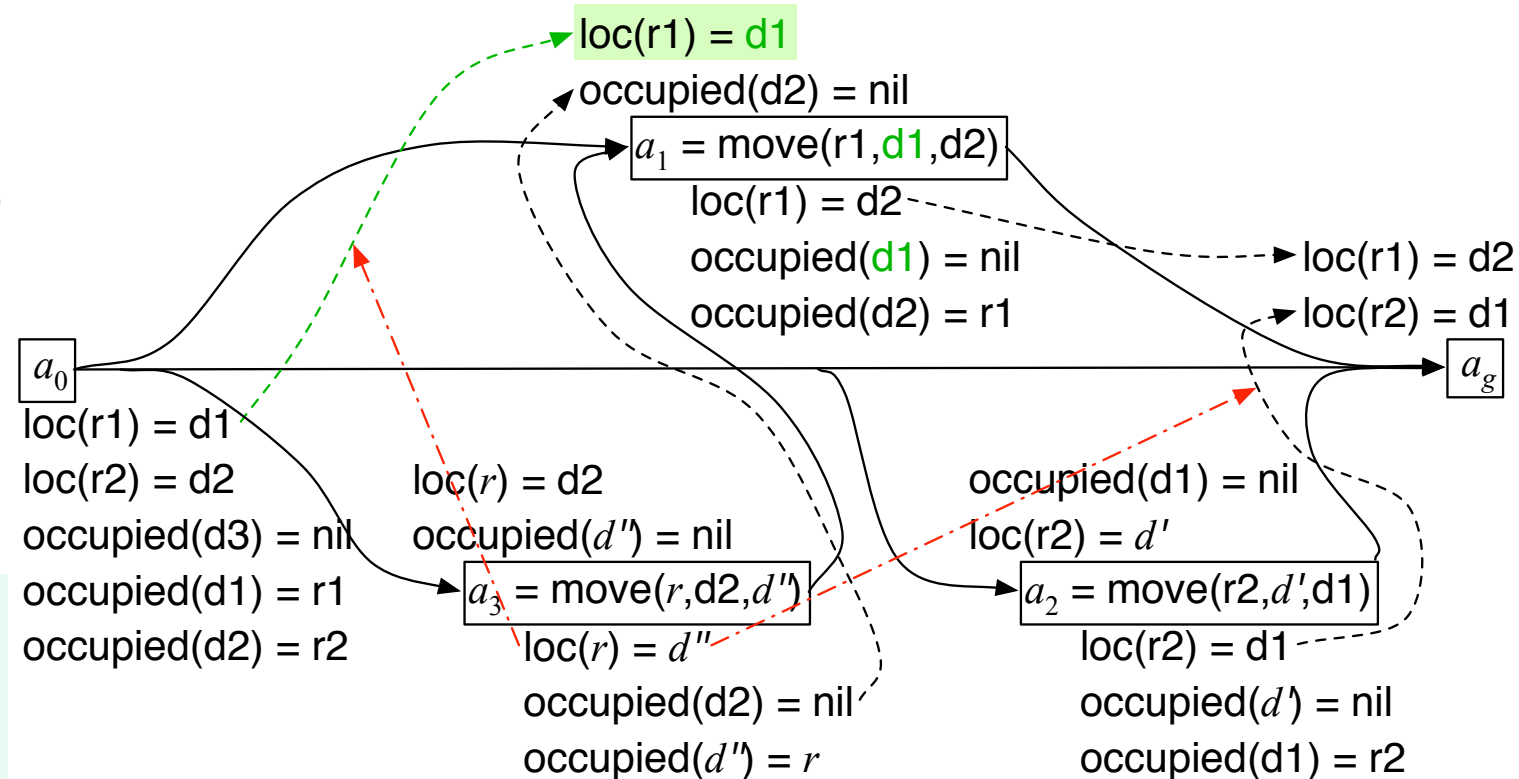
# Choosing a Resolver

**while**  $Flaws(\pi) \neq \emptyset$  **do**

- (i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return failure**
  - (ii) nondeterministically choose  $\rho \in R$   
modify  $\pi$  by applying  $\rho$  to it
- return**  $\pi$

Perhaps this might work:

- *Avoid New Actions (ANA)* heuristic:
  - ▶ prefer resolvers that don't add new actions
  - ▶ use LCR as tie-breaker



$\text{move}(r, d, d')$   
pre:  $loc(r) = d, occupied(d') = nil$   
eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$



PSP ( $\Sigma, \pi$ )

# Choosing a Resolver

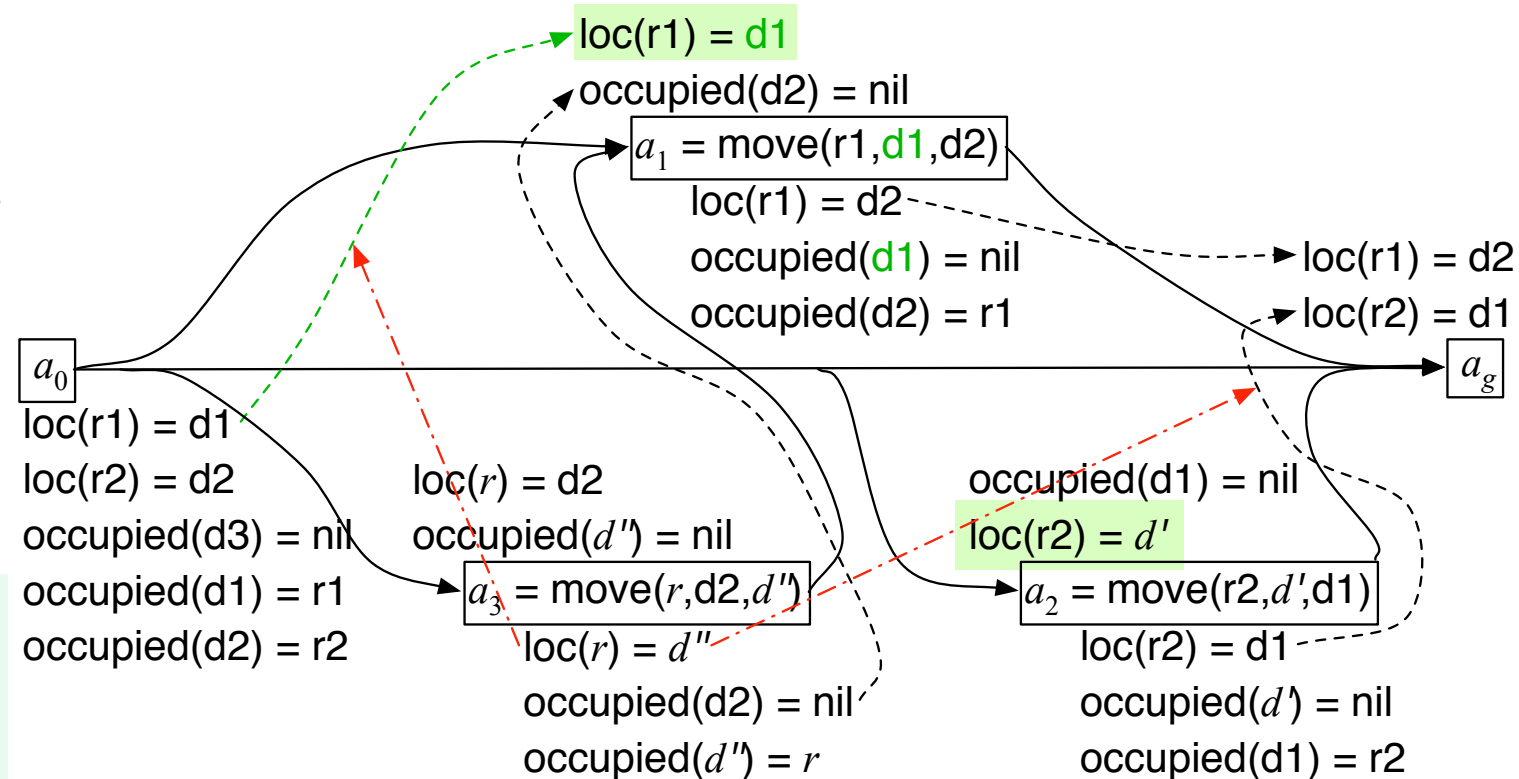
**while**  $Flaws(\pi) \neq \emptyset$  **do**

- (i) arbitrarily select  $f \in Flaws(\pi)$   
 $R \leftarrow \{\text{all feasible resolvers for } f\}$   
**if**  $R = \emptyset$  **then return failure**
  - (ii) nondeterministically choose  $\rho \in R$   
 modify  $\pi$  by applying  $\rho$  to it
- return**  $\pi$

- Problem: ANA will prefer these two choices:
  - ▶ For  $loc(r1)=d$  in  $a_1$ , use action  $a_0$  with substitution  $d \leftarrow d1$
  - ▶ For  $loc(r2)=d'$  in  $a_2$ , use action  $a_0$  with substitution  $d' \leftarrow d2$
  - ▶  $a_1 = \text{move}(r1,d1,d2)$ ;  $a_2 = \text{move}(r2,d2,d1)$ 
    - Makes the problem unsolvable  $\Rightarrow$  need to backtrack
- Perhaps use ANA anyway?

Perhaps this might work:

- *Avoid New Actions (ANA)* heuristic:
  - ▶ prefer resolvers that don't add new actions
  - ▶ use LCR as tie-breaker

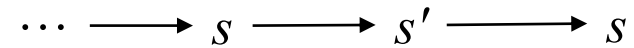


$\text{move}(r, d, d')$   
 pre:  $loc(r) = d, occupied(d') = nil$   
 eff:  $loc(r) \leftarrow d', occupied(d') = r, occupied(d) = nil$

# Discussion

- Problem: how to prune infinitely long paths in the search space?

- ▶ Loop detection is based on recognizing states or goals we've seen before
- ▶ Partially ordered plan: don't know the states



- Prune if  $\pi$  contains the same *action* more than once?

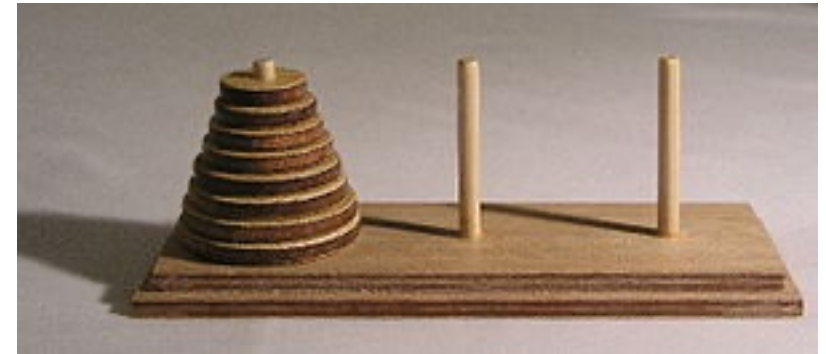
$\langle a_1, a_2, \dots, a_1, \dots \rangle$

- ▶ No. Sometimes need the same action again in another state
  - e.g., Towers of Hanoi: move disk1 from peg1 to peg2

- Weak pruning technique

- ▶ Prune all partial plans that contain more than  $|S|$  actions
- ▶ Not very helpful

- I don't know whether there's a better pruning technique



Credit: [Evanherk](#), [GFDL](#)

# Summary

- 3.3. Backward State-Space Search
  - ▶ Relevance,  $\gamma^{-1}$
  - ▶ Backward search, cycle checking
  - ▶ Lifted backward search (briefly)
- 3.4 Plan-Space Search
  - ▶ Definitions
    - Partially ordered plans and solutions
    - partial plans
    - causal links
  - ▶ flaws:
    - open goals, threats
  - ▶ resolvers
  - ▶ PSP algorithm
    - long example
    - brief discussion of node-selection heuristics, pruning techniques